

## EXERCISE 3B

1 Write down the terms in each series and then find the sum.

a  $\sum_{r=1}^4 2r + 1$

b  $\sum_{r=1}^3 r^2$

c  $\sum_{r=1}^5 r - 1$

d  $\sum_{r=2}^4 r(r + 1)$

e  $\sum_{r=2}^5 (r - 1)^2$

f  $\sum_{r=1}^4 \frac{1}{r}$

g  $\sum_{r=1}^6 (-1)^r r^2$

h  $\sum_{r=1}^7 4$

i  $\sum_{r=0}^5 2^r$

j  $\sum_{r=2}^5 (-1)^r 3r$

k  $\sum_{r=1}^5 7$

l  $\sum_{r=1}^3 \frac{r}{r + 1}$

2 Write each series using the  $\Sigma$  notation..

a  $1^2 + 2^2 + 3^2 + 4^2$

b  $3^1 + 3^2 + 3^3 + 3^4$

c  $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

d  $2 + 4 + 6 + 8$

e  $1 + 2 + 3 + 4 + \dots + 100$

f  $(1 \times 2) + (2 \times 3) + (3 \times 4) \dots n(n + 1)$

g  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots \frac{100}{101}$

h  $-1 + 1 - 1 + 1 - 1 + 1$

i  $5 + 9 + 13 + 17 + 21$

j  $4 + 9 + 14 + 19 + 24$

k  $1^3 + 2^3 + 3^3 + \dots$

l  $2 - 4 + 6 - 8 + 10 - \dots$

3 Work out  $\sum_{r=1}^5 2r - \sum_{r=1}^4 2r$

4 Work out  $\sum_{r=1}^{100} r^2 - \sum_{r=1}^{99} r^2$

5 Work out  $\sum_{r=10}^{1000} \frac{1}{r} - \sum_{r=10}^{999} \frac{1}{r}$

### Arithmetic series

Consider the *sequence* 3, 8, 13, 18, ...

If the terms of the sequence are added,

$$3 + 8 + 13 + 18$$

the expression is called a *series*.

In an arithmetic series the difference between consecutive terms is constant and is called the *common difference*.

Here are three different arithmetic series.

$$1 + 4 + 7 + 10 + \dots + 58,$$

$$4 + 14 + 24 + 34 + \dots + 214,$$

$$10 + 8 + 6 + \dots - 20,$$

Arithmetic series are sometimes called *arithmetic progressions* (A.P. for short).

The arithmetic series with first term  $a$  and common difference  $d$  is:

$$a + (a + d) + (a + 2d) + (a + 3d) + \dots$$

You can see that the  $n$ th term is  $a + (n - 1)d$