Substitute $y = -2x$ in $x^2 + xy + y^2 = 3$.

$$\begin{aligned} x^2 + x(-2x) + (-2x)^2 &= 3 \\ x^2 - 2x^2 + 4x^2 &= 3 \\ 3x^2 &= 3 \\ x^2 &= 1 \\ x &= 1 \text{ or } -1 \end{aligned}$$

From [1], when $x = 1, y = -2$
when $x = -1, y = 2$

The curve has stationary points at $(1, -2)$ and $(-1, 2)$.

[The second derivative will not be used here as the second derivative for implicit functions is not on the syllabus for C4.]

Exercise 1

1 For each curve find $\dfrac{dy}{dx}$ in terms of x and y.

a $x^2 + y^2 = 9$
b $xy = 4$
c $x^2 + xy + y^2 = 0$
d $x^2 + y^2 - 6x + 8y = 0$
e $x^2 + 3y^2 - 4y = 0$
f $y^3 = x^2 + 10$
g $3x^3 + 4xy^2 + y^3 = 0$

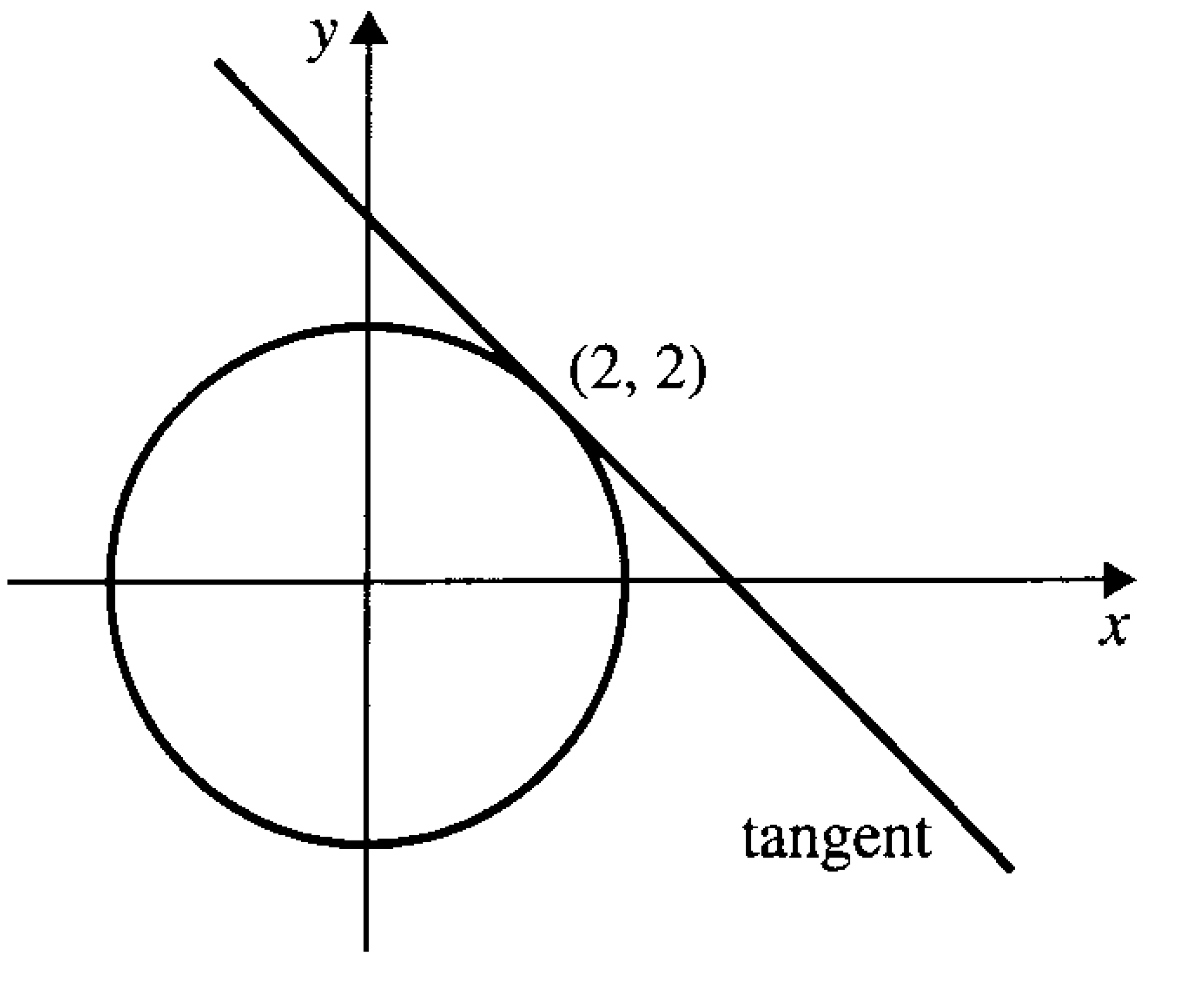
2 Find $\dfrac{dy}{dx}$ for the following curves at the points indicated:

a $x^2 + y^2 = 10 \quad (3, 1)$
b $xy = 9 \quad (3, 3)$
c $x^2 + y^2 + 4x = 9 \quad (1, 2)$
d $y^2 = x^3 \quad (4, 8)$

3 The diagram shows a circle with equation $x^2 + y^2 = 8$.

By finding $\dfrac{dy}{dx}$, calculate the gradient of the tangent to the circle at the point (2, 2).

Using the equation $y - y_1 = m(x - x_1)$, find the equation of the tangent at the point (2, 2).



4 Find the equation of the normal to $x^2 + y^2 = 8$ at the point (2, 2).
[Hint: Remember that gradient of tangent × gradient of normal = −1]

5 Find the equation of the tangent to the curve $xy = 4$ at the point (2, 2).

6 Find the equation of the normal to the curve $xy = 4$ at the point (−2, −2).

7 Find the equation of the tangent to the circle $x^2 + y^2 - 2x + 4y = 20$ at the point (4, 2).