Example 1

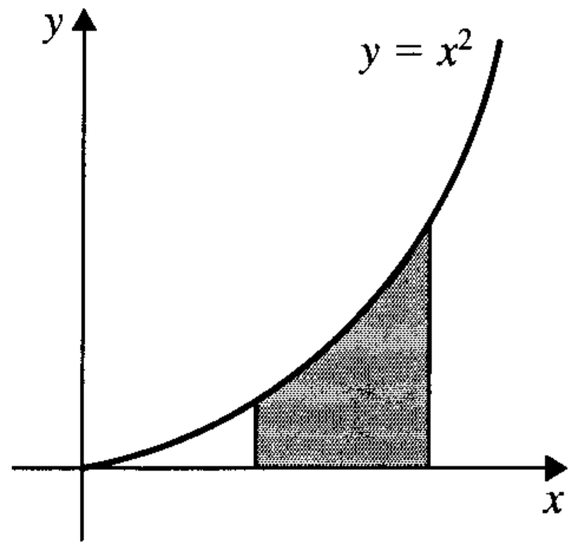
Find the volume of revolution formed by rotating the area enclosed by the curve $y = x^2$, the x-axis and the lines $x = 1$ and $x = 2$ through one revolution about the x-axis.

The volume formed, $V = \int_1^2 \pi y^2 \, dx$

$$= \pi \int_1^2 x^4 \, dx$$

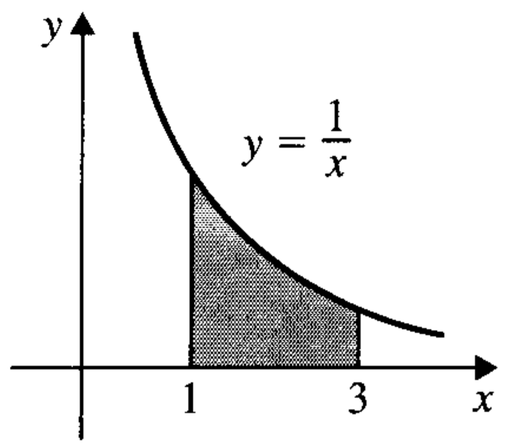
$$= \pi \left[\frac{x^5}{5} \right]_1^2$$

$$= \pi \left[\frac{32}{5} - \frac{1}{5} \right] = \frac{31}{5}\pi \text{ cubic units.}$$



EXERCISE 6

1 Find the volume of revolution when the area enclosed between the curve $y = \frac{1}{x}$, the x-axis and the lines $x = 1$ and $x = 3$ is rotated through 360° about the x-axis.



2 Find the volume of revolution when the area enclosed between the curve $y = \frac{1}{\sqrt{x}}$, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 360° about the x-axis.

3 Find the volume of revolution when the area enclosed between the curve $y = \sqrt{4 - x^2}$, the x-axis and the lines $x = -2$ and $x = 2$ is rotated through 360° about the x-axis.

In Questions 4 to 11 find the volume of revolution formed when the area enclosed between each curve, the x-axis and the values of x given is rotated through one revolution about the x-axis.

4 $y = x + 1$, between $x = 0$ and $x = 4$.

5 $y = \frac{x}{2}$, between $x = 2$ and $x = 4$.

6 $y = x^2 - 4$, between $x = -2$ and $x = 2$.

7 $y = 2x^{\frac{1}{4}}$, between $x = 1$ and $x = 16$.

8 $y = e^{2x}$, between $x = 0$ and $x = 1$.

9 $x^2 + y^2 = 9$, between $x = 0$ and $x = 3$.

10 $y = x^{\frac{1}{2}} e^x$, between $x = 1$ and $x = 2$.

11 $y = \frac{x}{\sqrt{x^3 + 4}}$, between $x = 0$ and $x = 2$.