Notice that in each case we have $y = \ln[f(x)]$

$$\text{and } \frac{dy}{dx} = \frac{1}{f(x)} \times f'(x).$$

This is a quick way of differentiating $\ln[f(x)]$.

Example 3

Find $\frac{dy}{dx}$ in each case.

a $y = (x^2 + 1)\ln 2x$

We have $y = uv$, where $u = x^2 + 1$ and $v = \ln 2x$.

Differentiating, $\frac{du}{dx} = 2x$, $\frac{dv}{dx} = \frac{1}{2x} \times 2 = \frac{1}{x}$.

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= (x^2 + 1)\frac{1}{x} + (\ln 2x)\,2x$$
$$= \frac{x^2 + 1}{x} + 2x\ln 2x$$

b $y = e^x \ln x^2$

We have $y = uv$, where $u = e^x$ and $v = \ln x^2$.

Differentiating, $\frac{du}{dx} = e^x$, $\frac{dv}{dx} = \frac{1}{x^2} \times 2x = \frac{2}{x}$.

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= e^x\frac{2}{x} + (\ln x^2)\,e^x$$
$$= \frac{2e^x}{x} + e^x\ln x^2$$

Exercise 5

1 Find $\frac{dy}{dx}$ for each of the following.

a $y = \ln 4x$ **b** $y = \ln x^3$ **c** $y = 6\ln x$

d $y = \ln(3x - 1)$ **e** $y = \ln(1 - 2x)$ **f** $y = \ln(x^3 + x)$

g $y = \ln\frac{x + 1}{2}$ **h** $y = \ln 4x^2$ **i** $y = 3\ln(x + 2)$

j $y = \ln\frac{1}{x}$ **k** $y = \ln\sqrt{x}$ **l** $y = \ln(x^2 + x - 2)$

2 **a** Write $\ln\left(\frac{x + 4}{x - 2}\right)$ as the difference of 2 logarithms.

b Hence find $\frac{d}{dx}\left[\ln\left(\frac{x + 4}{x - 2}\right)\right]$