Worked Solutions

AQA C1 Paper F

1. (a) $= 2^3(\sqrt{3})^3 = 8 \times 3\sqrt{3}$

$= 24\sqrt{3}$ *(1 mark)*

(b) $= 6 + \sqrt{2} - 3\sqrt{2} - 1 = 5 - 2\sqrt{2}$ *(1 mark)*

(c) $= \dfrac{\sqrt{2}(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$

$= \dfrac{2-\sqrt{2}}{1}$; $a = 2, b = -1$ *(3 marks)*

2. substitute for y in second equation

$x(7 - 3x) + 12 = 2x$

$3x^2 - 5x - 12 = 0$

$x = 3$ or $-\dfrac{4}{3}$

solutions are $(3, -2)$ and $\left(-\dfrac{4}{3}, 11\right)$ *(5 marks)*

3. line meets curve where $x^2 + 3x - 4 = 6$

$x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$, $x = 2$ or -5

$\dfrac{dy}{dx} = 2x + 3$

at $x = 2$, gradient $= 7$

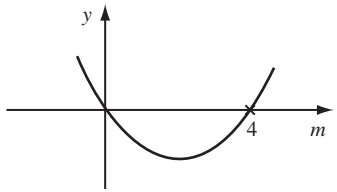
at $x = -5$, gradient $= -7$ *(7 marks)*

4. No real roots if '$b^2 < 4ac$'

$m^2 < 4m$

$m^2 - 4m < 0$ sketch $y = m^2 - 4m$

from graph $0 < m < 4$ *(5 marks)*



5. integrating $y = 2x^5 + 3x + c$

$y = 2, x = 1 : 2 = 2 + 3 + c$

$c = -3$

$\therefore y = 2x^5 + 3x - 3$

when $x = -1$, $y = 2(-1)^5 + 3(-1) - 3$

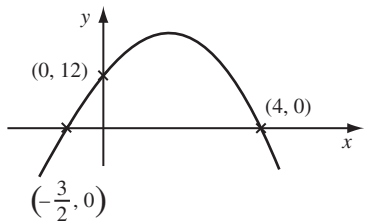
$y = -8$ *(6 marks)*

6. (a) $f(x) = (4 - x)(3 + 2x)$

$y = 0$ when $x = 4$ and $x = -\dfrac{3}{2}$

when $x = 0$, $y = 12$ *(3 marks)*

(b)



(2 marks)

(c) translation $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$ *(2 marks)*

7. (a) $f(1) = -4$,

$$1 + a + b - 4 = -4$$

$f(2) = 0$,

$$8 + 4a + 2b - 4 = 0$$

hence $a = -1, b = 0$ *(4 marks)*

(b) $(x - 2)\left(x^2 + x + 2\right)$ *(3 marks)*

(c) One root as $x^2 + x + 2 = 0$ has no real roots *(3 marks)*

8. (a) $f(x) = 0$

$$\Rightarrow \sqrt{x} - \frac{2}{\sqrt{x}} = 0$$

$$x - 2 = 0$$

$$x = 2 \quad \textit{(2 marks)}$$

(b) $f(x) = x\left(x - 2 - 2 + \frac{4}{x}\right)$

$$= x^2 - 4x + 4 \quad \textit{(3 marks)}$$

(c) $\int_{\frac{1}{2}}^{1} (x^2 - 4x + 4)\mathrm{d}x$

$$= \left[\frac{1}{3}x^3 - 2x^2 + 4x\right]_{\frac{1}{2}}^{1}$$

$$= \frac{1}{3} - 2 + 4 - \left(\frac{1}{3} \cdot \frac{1}{8} - 2 \cdot \frac{1}{4} + 2\right)$$

$$= \frac{19}{24} \quad \textit{(4 marks)}$$

9. (a) gradient of $AB = \frac{8 - 2}{3 - 1} = 3$ *(2 marks)*

(b) M is at $\left(\frac{3 + 1}{2}, \frac{8 + 2}{2}\right)$, i.e. $(2, 5)$

The line through C and M is the perpendicular bisector of AB

equation of CM is $y - 5 = -\frac{1}{3}(x - 2)$

$$3y + x = 17 \quad \ldots [A] \quad \textit{(5 marks)}$$

(c) at C, $y = 4$

substitute in equation $[A]$, $12 + x = 17$

$$x = 5 \quad \textit{(2 marks)}$$

(d) C is $(5, 4)$, A is $(1, 2)$

radius $= \sqrt{(5 - 1)^2 + (4 - 2)^2}$

$$= \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5} \quad \textit{(4 marks)}$$

10. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 1$

at P, $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$

$\therefore$ gradient of normal $= -1$ and when $x = 1$, $y = 0$

equation of normal is $y - 0 = -1(x - 1)$

$$y = -x + 1 \quad \textit{(6 marks)}$$

(b) normal intersects curve where $x^2 - x = -x + 1$

i.e. $x^2 = 1$

$x = 1$ (at P)

and $x = -1$

coordinates of second point are $(-1, 2)$ *(4 marks)*