

Core Mathematics C3 Advanced Level

For OCR

Paper D

Time: 1 hour 30 minutes

Instructions and Information

- Full marks may be obtained for answers to ALL questions.
- The booklet 'List of Formulae MFI', available from OCR, may be used.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You may use a graphical calculator in this paper.
- The total number of marks for this paper is 72.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. (i) Simplify $\frac{x^2 + 7x + 12}{x^2 + 3x}$. (2)

(ii) Find the value of x for which

$$\log_3(x^2 + 7x + 12) - \log_3(x^2 + 3x) = 2. \quad (4)$$

2. The functions f and g are defined by

$$f: x \mapsto \frac{1}{x-1}, \quad x \in \mathbb{R}, \quad x > 1$$

$$g: x \mapsto 3^x + k, \quad x \in \mathbb{R} \text{ and } k \text{ is a constant.}$$

(i) Find the range of f . (2)

(ii) Given that $gf\left(\frac{3}{2}\right) = 14$, find the value of k . (4)

3. (i) Prove that

$$\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1. \quad (4)$$

(ii) Using part (a) solve the equation

$$8 \cos^2 x - 8 \cos^4 x = 1, \quad \text{for } 0 < x < 180^\circ. \quad (3)$$

4. (a) Differentiate with respect to x

(i) $x^3 e^{3x}$ (3)

(ii) $\frac{2x}{\cos x}$ (3)

(b) Given that $x = \cos y^2$,

find $\frac{dy}{dx}$ in terms of y . (4)

5. (i) Sketch on the same set of axes the graphs of

$$y = \ln(x - 3) \quad x \in \mathbb{R}, \quad x > 3.$$

$$\text{and } y = \frac{2}{x}, \quad x \neq 0 \quad (3)$$

Given that $f(x) = \ln(x - 3) - \frac{2}{x}$, $x > 3$,

- (ii) Explain how your graphs show that the equation $f(x) = 0$ has only one solution. (1)

- (iii) Show that the solution of $f(x) = 0$ lies between $x = 4$ and $x = 5$. (2)

- (iv) The iterative formula

$$x_{n+1} = e^{\frac{2}{x_n}} + 3$$

is used to solve the equation $f(x) = 0$.

Taking $x_0 = 5$, find the values of x_1, x_2, x_3 and x_4 and hence find an approximation to the solution of $f(x) = 0$, giving your answer to 2 decimal places. (3)

6. (i) Given that $\cot x = \frac{4}{3}$ and that x is a reflex angle, find the exact value of $\operatorname{cosec} x$. (3)

- (ii) Find the exact value of $\cos 2x$. (3)

- (iii) Find the exact value of $\tan 2x$. (3)

7. (i) Sketch on the same pair of axes the graphs of

$$y = |x^2 - a^2| \quad \text{and} \quad y = x + a; \quad a > 1$$

Mark clearly on your graphs the coordinates of the points of intersection with the x and y axes. (4)

- (ii) The curve and straight line intersect at three points.

Show that one of these points has coordinates $(-a, 0)$. (1)

- (iii) Show that the x coordinate of one of the other points satisfies the equation

$$x^2 - x - a^2 - a = 0. \quad (2)$$

- (iv) Write down an equation which is satisfied by the third point of intersection. (2)

- (v) Given that $a = 3$, find the coordinates of the points of intersection of the curve and the straight line. (2)

8. (i) Show that $x = e^{-t}(\cos t + \sin t)$ satisfies the equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0. \quad (6)$$

- (ii) Find the value of x , when $t = 0$, and show that $\frac{dx}{dt} = 0$, when $t = 0$. (2)

- (iii) Find the next positive value for t for which $\frac{dx}{dt} = 0$. (2)

- (iv) Hence find a maximum and minimum value for x . (2)

- (v) Justify your answers to (iv) by evaluating $\frac{d^2x}{dt^2}$. (2)