

# Core Mathematics C3 Advanced Level

# For OCR

## Paper D

**Time: 1 hour 30 minutes**

### *Instructions and Information*

- Full marks may be obtained for answers to ALL questions.
- The booklet 'List of Formulae MFI', available from OCR, may be used.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You may use a graphical calculator in this paper.
- The total number of marks for this paper is 72.

### *Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner.  
Answers without working may gain no credit.

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1. (i) Simplify  $\frac{x^2 + 7x + 12}{x^2 + 3x}$ . (2)

(ii) Find the value of  $x$  for which

$$\log_3(x^2 + 7x + 12) - \log_3(x^2 + 3x) = 2. \quad (4)$$

2. The functions  $f$  and  $g$  are defined by

$$f: x \mapsto \frac{1}{x-1}, \quad x \in \mathbb{R}, \quad x > 1$$

$$g: x \mapsto 3^x + k, \quad x \in \mathbb{R} \text{ and } k \text{ is a constant.}$$

(i) Find the range of  $f$ . (2)

(ii) Given that  $gf\left(\frac{3}{2}\right) = 14$ , find the value of  $k$ . (4)

3. (i) Prove that

$$\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1. \quad (4)$$

(ii) Using part (a) solve the equation

$$8 \cos^2 x - 8 \cos^4 x = 1, \quad \text{for } 0 < x < 180^\circ. \quad (3)$$

4. (a) Differentiate with respect to  $x$

(i)  $x^3 e^{3x}$  (3)

(ii)  $\frac{2x}{\cos x}$  (3)

(b) Given that  $x = \cos y^2$ ,

find  $\frac{dy}{dx}$  in terms of  $y$ . (4)

5. (i) Sketch on the same set of axes the graphs of

$$y = \ln(x - 3) \quad x \in \mathbb{R}, \quad x > 3.$$

$$\text{and } y = \frac{2}{x}, \quad x \neq 0 \quad (3)$$

Given that  $f(x) = \ln(x - 3) - \frac{2}{x}$ ,  $x > 3$ ,

- (ii) Explain how your graphs show that the equation  $f(x) = 0$  has only one solution. (1)

- (iii) Show that the solution of  $f(x) = 0$  lies between  $x = 4$  and  $x = 5$ . (2)

- (iv) The iterative formula

$$x_{n+1} = e^{\frac{2}{x_n}} + 3$$

is used to solve the equation  $f(x) = 0$ .

Taking  $x_0 = 5$ , find the values of  $x_1, x_2, x_3$  and  $x_4$  and hence find an approximation to the solution of  $f(x) = 0$ , giving your answer to 2 decimal places. (3)

6. (i) Given that  $\cot x = \frac{4}{3}$  and that  $x$  is a reflex angle, find the exact value of  $\operatorname{cosec} x$ . (3)

- (ii) Find the exact value of  $\cos 2x$ . (3)

- (iii) Find the exact value of  $\tan 2x$ . (3)

7. (i) Sketch on the same pair of axes the graphs of

$$y = |x^2 - a^2| \quad \text{and} \quad y = x + a; \quad a > 1$$

Mark clearly on your graphs the coordinates of the points of intersection with the  $x$  and  $y$  axes. (4)

- (ii) The curve and straight line intersect at three points.

Show that one of these points has coordinates  $(-a, 0)$ . (1)

- (iii) Show that the  $x$  coordinate of one of the other points satisfies the equation

$$x^2 - x - a^2 - a = 0. \quad (2)$$

- (iv) Write down an equation which is satisfied by the third point of intersection. (2)

- (v) Given that  $a = 3$ , find the coordinates of the points of intersection of the curve and the straight line. (2)

8. (i) Show that  $x = e^{-t}(\cos t + \sin t)$  satisfies the equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0. \quad (6)$$

- (ii) Find the value of  $x$ , when  $t = 0$ , and show that  $\frac{dx}{dt} = 0$ , when  $t = 0$ . (2)

- (iii) Find the next positive value for  $t$  for which  $\frac{dx}{dt} = 0$ . (2)

- (iv) Hence find a maximum and minimum value for  $x$ . (2)

- (v) Justify your answers to (iv) by evaluating  $\frac{d^2x}{dt^2}$ . (2)