Core Mathematics C4 For AQA
Advanced Level

Paper H

Time: 1 hour 30 minutes

Instructions and Information

- Full marks may be obtained for answers to ALL questions.
- The formulae booklet, available from AQA, may be used.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You may use a graphical calculator in this paper.
- The total number of marks for this paper is 75.

Advice to Candidates
You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. Use the substitution $u = e^x$ to show that the value of

$$\int_{-1}^{1} \frac{1}{1 + e^{-x}}\, dx \text{ is } 1. \qquad \textit{(4 marks)}$$

2. (a) The equation of a curve is

$$x = e^y.$$

(i) Find an expression for $\frac{dy}{dx}$ in terms of x. *(2 marks)*

(ii) Find the equation of the tangent to the curve at the point where $y = 0$. *(2 marks)*

(b) For the curve $x = \sin y$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$. *(3 marks)*

3. A curve has parametric equations

$$x = 2\sin\theta + 1, \quad y = 2\cos\theta + 2.$$

(a) Show that the equation of the tangent at the point with parameter θ is

$$x\sin\theta + y\cos\theta = 2 + 2\cos\theta + \sin\theta \qquad \textit{(4 marks)}$$

(b) Write down the equation of the tangent at the point where $\theta = \frac{\pi}{2}$. *(1 mark)*

(c) Find the cartesian equation of the curve. *(4 marks)*

4. Points on a curve C satisfy the differential equation

$$\frac{dy}{dx} = -\frac{x - 2}{y + 1}.$$

The point (2, 2) lies on C.

(a) Show that the equation of C may be written as

$$(x - 2)^2 + (y + 1)^2 = 9. \qquad \textit{(6 marks)}$$

(b) Sketch the curve C. *(2 marks)*

5. (a) Prove by counter-example, that the statement

"$\operatorname{cosec}(A + B) \equiv \operatorname{cosec} A + \operatorname{cosec} B$ for all A and B" is false. *(2 marks)*

(b) Prove that

$$2 \operatorname{cosec} 2A \equiv \sec A \operatorname{cosec} A, \quad A \neq \frac{n\pi}{2}, n \in \mathbb{Z}. \qquad \textit{(3 marks)}$$

6. (a) Express in the form $Ax + B$, where A and B are to be found,

$$\frac{2x^2 - x - 3}{x + 1} + \frac{x^2 - 4}{x + 2}. \qquad \textit{(4 marks)}$$

(b) Hence solve the equation

$$\frac{2x^2 - x - 3}{x + 1} + \frac{x^2 - 4}{x + 2} = x^2 - 9 \qquad \textit{(2 marks)}$$

7. A warm object is immersed in a cold liquid. At time t minutes its temperature $\theta^\circ C$ is given by

$$\theta = 70\,\mathrm{e}^{-0.1t} + 2.$$

(a) Write down the initial value of θ. *(1 mark)*

(b) Find the value of θ when $t = 10$. *(2 marks)*

(c) State the value which the temperature of the object approaches after a long time. *(2 marks)*

(d) Find the time taken for the temperature of the object to reach $10^\circ C$. *(3 marks)*

8. (a) Express

$$\frac{9x}{(1 - 2x)(1 + x)^2}$$

in partial fractions. *(4 marks)*

(b) Hence, or otherwise, find the expansion of $\dfrac{5x}{(1 - 2x)(1 + x)^2}$ as a series in ascending powers of x as far as the term in x^3. *(5 marks)*

9. (a) Express

$$2.5\sin 2x + 6\cos 2x \qquad \text{in the form}$$

$R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving your values of R and α to 3 decimal places where appropriate. *(4 marks)*

(b) Express $5\sin x\cos x - 12\sin^2 x$ in the form

$a\cos 2x + b\sin 2x + c$, where a, b and c are constants to be found. *(4 marks)*

(c) Hence, using your answer to part (*a*), deduce the maximum value of

$$5\sin x\cos x - 12\sin^2 x. \qquad \textit{(2 marks)}$$

10. Points A and B have position vectors $\begin{pmatrix} 7 \\ 8 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ 7 \\ 3 \end{pmatrix}$ respectively, relative to an origin O.

(a) Find a vector equation of the line through A and B in terms of a parameter λ. *(3 marks)*

(b) Calculate the acute angle between OA and AB, correct to the nearest degree. *(2 marks)*

(c) The point M on AB is such that OM is perpendicular to AB. Find the position vector of M. *(4 marks)*