# Worked Solutions

## AQA C4 Paper H

**1.** (a) let $I = \int_{-1}^{1} \frac{1}{1+e^{-x}}dx$

put $u = e^x$

$\frac{du}{dx} = e^x$

$du = e^x \ dx$

$= \int_{-1}^{1} \frac{e^x}{e^x+1}dx$

when $x = 1, \quad u = e$

$x = -1, \quad u = e^{-1}$

$\therefore \quad I = \int_{e^{-1}}^{e} \frac{du}{u+1} = \Big[\ln(u+1)\Big]_{e^{-1}}^{e}$

$= \ln(e+1) - \ln\left(1+\frac{1}{e}\right)$

$= \ln\left(\frac{e+1}{1+\frac{1}{e}}\right) = \ln\left[\frac{(1+e)e}{(e+1)}\right] = \ln e = 1$ *(4 marks)*

**2.** (a) (i) differentiating implicitly, $1 = e^y\frac{dy}{dx}$ $\quad \frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$ *(2 marks)*

(ii) when $y = 0, \ x = e^0 = 1$ $\quad \frac{dy}{dx} = 1$

equation of tangent is $y - 0 = x - 1$

$y = x - 1$ *(2 marks)*

(b) $x = \sin y$ $\quad 1 = \cos y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$ *(3 marks)*

**3.** (a) $\frac{dy}{dx} = \frac{-2\sin\theta}{2\cos\theta} = -\frac{\sin\theta}{\cos\theta}$

equation of tangent is $y - (2\cos\theta + 2) = -\frac{\sin\theta}{\cos\theta}\Big[x - (2\sin\theta + 1)\Big]$

$y\cos\theta - 2\cos^2\theta - 2\cos\theta = -x\sin\theta + 2\sin^2\theta + \sin\theta$

$x\sin\theta + y\cos\theta = 2 + 2\cos\theta + \sin\theta$ *(4 marks)*

(b) when $\theta = \frac{\pi}{2}$ tangent is $x + 0 = 2 + 0 + 1$

$x = 3$ *(1 mark)*

(c) $\sin\theta = \frac{x-1}{2}, \cos\theta = \frac{y-2}{2}$

$\left(\frac{x-1}{2}\right)^2 + \left(\frac{y-2}{2}\right)^2 = 1$ $\quad \Big[\sin^2\theta + \cos^2\theta = 1\Big]$

$(x-1)^2 + (y-2)^2 = 4$ *(4 marks)*

**4.** (a) $\int(y+1)dy = -\int(x-2)dx$

$\frac{1}{2}(y+1)^2 = -\frac{1}{2}(x-2)^2 + k$

$(2, 2)$ lies on $C$,

$\therefore \quad \frac{1}{2}9 = -\frac{1}{2}\times 0 + C$

$C$ is $\frac{1}{2}(y+1)^2 + \frac{1}{2}(x-2)^2 = \frac{9}{2}$

or $\quad (x-2)^2 + (y+1)^2 = 9$ *(6 marks)*

(b) Circle centre $(2, -1)$, radius 3 *(2 marks)*

**5.** (a) any valid pair of $A \& B$ e.g. $A = B = \pi/2$ *(2 marks)*

(b) L.H.S. $= \dfrac{2}{\sin 2A} = \dfrac{2}{2\sin A\cos A} = \operatorname{cosec} A\sec A$ *(3 marks)*

**6.** (a) $\dfrac{(2x-3)\cancel{(x+1)}}{\cancel{(x+1)}} + \dfrac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}$

$= 3x - 5 \Rightarrow A = 3,\ B = -5$ *(4 marks)*

(b) $3x - 5 = x^2 - 9$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$ *(2 marks)*

**7.** (a) $t = 0, \theta = 70 + 2 = 72$ *(1 mark)*

(b) $\theta = 70\,\mathrm{e}^{-1} + 2 = 27.8$ *(2 marks)*

(c) as $t \to \infty, \mathrm{e}^{-0.1t} \to 0$

$\therefore \quad \theta \to 2$ *(2 marks)*

(d) $10 = 70\,\mathrm{e}^{-0.1t} + 2$

$\mathrm{e}^{-0.1t} = \dfrac{8}{70}$

$-0.1t = \ln\dfrac{8}{70}, \quad t = 21.7$ minutes *(3 marks)*

**8.** (a) $\dfrac{9x}{(1-2x)(1+x)^2} \equiv \dfrac{A}{(1-2x)} + \dfrac{B}{(1+x)} + \dfrac{C}{(1+x)^2}$

$9x \equiv A(1+x)^2 + B(1-2x)(1+x) + C(1-2x)$

$x = \dfrac{1}{2}$: $\quad \dfrac{9}{2} = A\cdot\left(\dfrac{3}{2}\right)^2 \Rightarrow A = 2$

$x = -1$ : $\quad -9 = C(1+2) \Rightarrow C = -3$

constants: $\quad 0 = A + B + C \Rightarrow B = 1$

$\therefore$ expression is $\dfrac{2}{1-2x} + \dfrac{1}{1+x} - \dfrac{3}{(1+x)^2}$ *(4 marks)*

(b) $2(1-2x)^{-1} + (1+x)^{-1} - 3(1+x)^{-2}$

$$= 2\left[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}\left(-2x\right)^2 + \frac{(-1)(-2)(-3)}{3.2}\left(-2x\right)^3\right]$$

$$+\left[1 - x + \frac{(-1)(-2)}{2}\left(x^2\right) + \frac{(-1)(-2)(-3)}{3.2}x^3\right]$$

$$-3\left[1 + (-2)x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{3.2}x^3\right]$$

$$= \left(2 + 4x + 8x^2 + 16x^3\right) + \left(1 - x + x^2 - x^3\right) - 3\left(1 - 2x + 3x^2 - 4x^3\right)$$

$= 9x + 27x^3$ *(5 marks)*

**9.** (a) $R^2 = 2.5^2 + 6^2$

$R = 6.5$

$\tan\alpha = \dfrac{6}{2.5}$

$\alpha = 1.176^c$ radians *(4 marks)*

(b) $5\sin x\cos x - 12\sin^2 x$

$= 2.5(\sin 2x) + 6\left(-2\sin^2 x + 1\right) - 6$

$= 2.5\sin 2x + 6\cos 2x - 6$ *(4 marks)*

(c) $= 6.5\sin(2x + 1.176) - 6$

maximum value $6.5 - 6 = 0.5$ *(2 marks)*

**10.** (a) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, line through $AB$ is $\mathbf{r} = \begin{pmatrix} 7 \\ 8 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ *(3 marks)*

(b) $\overrightarrow{AO} = \begin{pmatrix} -7 \\ -8 \\ 0 \end{pmatrix}$ $\quad \left|\overrightarrow{AO}\right| = \sqrt{49 + 64} = \sqrt{113}$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ $\quad \left|\overrightarrow{AB}\right| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$\begin{pmatrix} -7 \\ -8 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \sqrt{113}\ \sqrt{14}\cos\theta$, where $\theta =$ angle required

$-14 + 8 = \sqrt{113}\ \sqrt{14}\cos\theta$

$\theta = 98.7°$

acute angle between $OA$ and $AB$ is $81°$ (nearest degree)

(c) $M$ lies on line $AB$

$\therefore \quad \overrightarrow{OM}$ is $\begin{pmatrix} 7 + 2\lambda \\ 8 - \lambda \\ 0 + 3\lambda \end{pmatrix}$

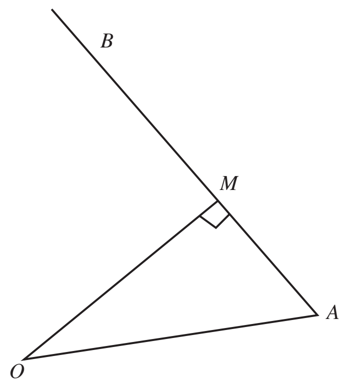
$\overrightarrow{OM} \cdot \overrightarrow{AB} = 0$

$\begin{pmatrix} 7 + 2\lambda \\ 8 - \lambda \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$= 14 + 4\lambda - 8 + \lambda + 9\lambda$

$= 0$

$\lambda = \dfrac{-3}{7}$




position vector of $M$ is $\begin{pmatrix} 7 - \frac{6}{7} \\ 8 + \frac{3}{7} \\ \frac{-9}{7} \end{pmatrix} = \dfrac{1}{7}\begin{pmatrix} 43 \\ 59 \\ -9 \end{pmatrix}$ *(4 marks)*