**1.** $$f(x) = \frac{3x}{(x-1)(x+2)}.$$

($a$) Express $f(x)$ in partial fractions. **(3)**

($b$) Evaluate $\int_{-1}^{0} f(x)\,dx$. **(4)**

---

**2.** A curve has parametric equations

$$x = 1 - t^3,\; y = 1 + t^2.$$

($a$) Find the value of the parameter $t$ at the point (2, 2). **(1)**

($b$) Find the equation of the tangent to the curve at (2, 2). **(4)**

---

**3.**

**Figure 1**

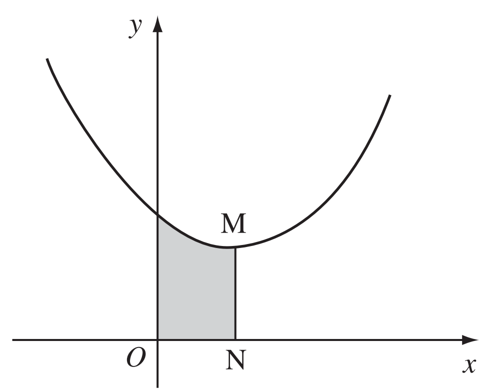


Figure 1 shows the curve with equation

$$y = e^x - 3x.$$

The minimum point on the curve is $M$ and the line $MN$ is parallel to the $y$-axis.

($a$) Find the $x$-coordinate of $M$. **(2)**

($b$) Show that the area of the shaded region can be written as

$$a - b(\ln 3)^2,$$

where the constants $a$ and $b$ are to be determined. **(5)**

---

**4.** A curve has equation

$$4x^2 + 3y^2 - 2xy = 32.$$

(*a*) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$, simplifying your answer. **(6)**

(*b*) Find the gradient of the curve at the point (2, 4) and hence find the equation of a tangent to the curve at that point. **(3)**

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**5.** In the series expansion of $(1 + kx)^n$, the coefficients of $x$ and $x^2$ are $-6$ and 27 respectively. Find

(*a*) the value of $k$ and the value of $n$, **(4)**

(*b*) the coefficient of $x^3$ in the expansion, **(3)**

(*c*) the set of values of $x$ for which the expansion is valid. **(1)**

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**6.** (*a*) Solve the differential equation

$$x^2\frac{\mathrm{d}y}{\mathrm{d}x} = y^2(4x^5 - 1),$$

given that $y = \dfrac{1}{2}$ when $x = 1$. **(6)**

(*b*) Use the substitution $t = 1 + x^2$ to show that

$$\int_0^2 \frac{x^3}{(1 + x^2)^{\frac{1}{2}}}\,\mathrm{d}x = \frac{2}{3}(1 + \sqrt{5}).$$

**(7)**

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**7.**

**Figure 2**

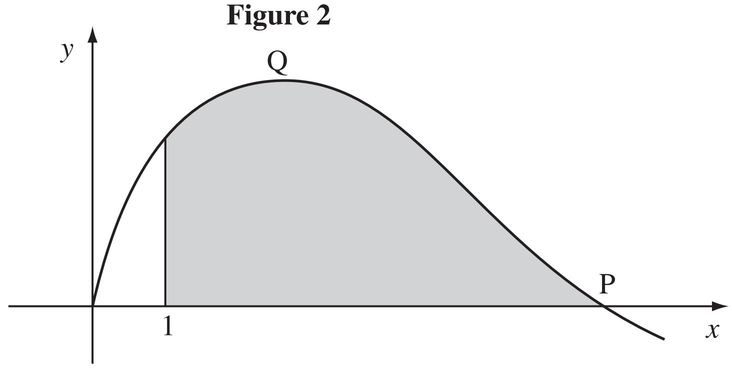


Figure 2 shows the graph of $y = 2x - x \ln x$. The graph crosses the $x$-axis at the point $P$ and has a turning point at $Q$.

($a$) Find the coordinates of $Q$.

Verify that $\frac{d^2y}{dx^2} < 0$ at this point. **(4)**

($b$) Show that the coordinates of $P$ are $(e^2, 0)$. **(2)**

($c$) (i) Show that $\int_1^{e^2} x \ln x \, dx = \frac{3e^4 + 1}{4}$. **(6)**

(ii) Find the area of the shaded region bounded by the curve, the $x$-axis and the line $x = 1$. **(3)**

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**8.** The position vectors of $A, B, C$ and $D$ with respect to the origin are:

$$A \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} \quad B \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \quad C \begin{pmatrix} 9 \\ 3 \\ 0 \end{pmatrix} \quad D \begin{pmatrix} 4 \\ 8 \\ 2 \end{pmatrix}$$

($a$) The line through $B$ and $C$ is denoted by $l_1$ and the line through $A$ and $D$ is denoted by $l_2$. Show that $l_1$ has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix}$$ **(2)**

($b$) Find an equation for $l_2$. **(2)**

($c$) Find the position vector of the point where $l_1$ and $l_2$ intersect. **(4)**

($d$) Calculate the acute angle between $l_1$ and $l_2$, correct to one decimal place. **(3)**

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**END** **TOTAL 75 MARKS**