

Worked Solutions

Edexcel C4 Paper A

1. (a) $\frac{3x}{(x-1)(x+2)} = \frac{1}{x-1} + \frac{2}{x+2}$ (using 'cover up' rule) (3)

(b)
$$\int_2^3 \left(\frac{1}{x-1} + \frac{2}{x+2} \right) dx = \left[\ln(x-1) + 2\ln(x+2) \right]_2^3$$

$$= \ln 2 + 2\ln 5 - (\ln 1 + 2\ln 4)$$

$$= \ln 2 + 2\ln \frac{5}{4}$$

$$= \ln 2 + \ln \frac{25}{16}$$

$$= \ln \frac{25}{8}$$
 (4)

2. (a) $x = 1 - t^3$ and $x = 2$

$$\therefore 1 - t^3 = 2$$

$$t^3 = -1$$

$$t = -1$$
 (1)

(b) $\frac{dy}{dt} = 2t, \quad \frac{dx}{dt} = -3t^2$

$$\frac{dy}{dx} = \frac{2t}{-3t^2} = \frac{-2}{3t}$$

when $t = -1$, gradient of tangent = $\frac{2}{3}$.

equation of tangent is $y - 2 = \frac{2}{3}(x - 2)$

$$3y = 2x + 2$$
 (4)

3. (a) $\frac{dy}{dx} = e^x - 3$

at M $e^x = 3$

$$x = \ln 3$$
 (2)

(b) area =
$$\int_0^{\ln 3} (e^x - 3x) dx = \left[e^x - \frac{3}{2}x^2 \right]_0^{\ln 3}$$

$$= e^{\ln 3} - \frac{3}{2}(\ln 3)^2 - (1 - 0)$$

$$= 3 - \frac{3}{2}(\ln 3)^2 - 1$$

$$= 2 - \frac{3}{2}(\ln 3)^2$$
 (5)

4. (a) $8x + 6y \frac{dy}{dx} - \left(2x \frac{dy}{dx} + y \cdot 2 \right) = 0$

$$\frac{dy}{dx}(6y - 2x) = 2y - 8x$$

$$\frac{dy}{dx} = \frac{2y - 8x}{6y - 2x} = \frac{y - 4x}{3y - x}$$
 (6)

(b) at (2, 4), $\frac{dy}{dx} = \frac{4 - 8}{12 - 2} = -\frac{2}{5}$

equation of tangent at (2, 4) is

$$y - 4 = -\frac{2}{5}(x - 2)$$

$$5y - 20 = -2x + 4$$

$$5y + 2x = 24$$
 (3)

$$5. (a) (1+kx)^n = 1+nkx + \frac{n(n-1)}{2} \cdot k^2x^2 + \frac{n(n-1)(n-2)}{3 \cdot 2} \cdot k^3x^3 + \dots$$

$$nk = -6 \quad \dots[A]$$

$$\frac{n(n-1)}{2}k^2 = 27 \quad \dots[B]$$

$$\text{from [A]} \quad k = \frac{-6}{n}$$

$$\text{substitute in [B]} \quad \frac{n(n-1)}{2} \left(\frac{-6}{n}\right)^2 = 27$$

$$\text{hence } n = -2 \quad \text{and} \quad k = 3 \quad (4)$$

$$(b) \text{ coef. of } x^3 = \frac{-2 \cdot -3 \cdot -4}{3 \cdot 2} \cdot 27 = -108 \quad (3)$$

$$(c) \text{ valid for } -1 < 3x < 1$$

$$\text{i.e. } -\frac{1}{3} < x < \frac{1}{3} \quad (1)$$

$$6. (a) \text{ Separating the variables, } y^{-2}dy = \frac{4x^5 - 1}{x^2}dx$$

$$\int y^{-2} dy = \int (4x^3 - x^{-2}) dx$$

$$-\frac{1}{y} = x^4 + \frac{1}{x} + c$$

$$y = \frac{1}{2}, x = 1 \Rightarrow -2 = 1 + 1 + c \quad c = -4$$

$$\therefore -\frac{1}{y} = x^4 + \frac{1}{x} - 4 \quad (7)$$

$$\text{or } -\frac{1}{y} = \frac{x^5 + 1 - 4x}{x}$$

$$y = -\left(\frac{x}{x^5 + 1 - 4x}\right)$$

$$(b) \text{ Let } I = \int_0^2 \frac{x^3}{(1+x^2)^{\frac{1}{2}}} dx$$

$$\text{Let } t = 1 + x^2 \\ \frac{dt}{dx} = 2x$$

$$\frac{1}{2} dt = x dx$$

$$\therefore I = \frac{1}{2} \int_1^5 \frac{(t-1)dt}{t^{\frac{1}{2}}}$$

$$\text{when } x = 0, \quad t = 1 \\ x = 2, \quad t = 5$$

$$= \frac{1}{2} \int_1^5 \left(t^{\frac{1}{2}} - t^{-\frac{1}{2}}\right) dt$$

$$= \frac{1}{2} \left[\frac{2}{3} t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right]_1^5 = \frac{1}{2} \left[\frac{2}{3} \cdot 5\sqrt{5} - 2\sqrt{5} - \left(\frac{2}{3} - 2\right) \right]$$

$$= \frac{1}{2} \left[\frac{10\sqrt{5} - 6\sqrt{5} - 2 + 6}{3} \right]$$

$$= \frac{1}{6} [4\sqrt{5} + 4] = \frac{2}{3} (1 + \sqrt{5}) \quad (7)$$

$$7. (a) \frac{dy}{dx} = 2 - \left(x \cdot \frac{1}{x} + \ln x\right) = 1 - \ln x$$

$$\text{at } Q \quad \frac{dy}{dx} = 0 \quad \therefore \ln x = 1 \quad x = e$$

$$\text{at } x = e, \quad y = 2e - e \ln e = e$$

$$Q \text{ is at } (e, e)$$

$$\frac{d^2y}{dx^2} = -\frac{1}{x}, \quad \text{so } \frac{d^2y}{dx^2} < 0 \quad \text{at } x = e \quad (4)$$

$$(b) \text{ at } P \quad 2x - x \ln x = 0$$

$$x(2 - \ln x) = 0$$

$$\ln x = 2$$

$$x = e^2$$

$$\text{coordinates of } P \text{ are } (e^2, 0) \quad (2)$$

$$\begin{aligned}
(c) \text{ (i)} \int_1^{e^2} x \ln x \, dx &= \int_1^{e^2} \ln x \frac{d}{dx} \left(\frac{1}{2} x^2 \right) dx && \text{[By parts]} \\
&= \left[\frac{1}{2} x^2 \ln x \right]_1^{e^2} - \int_1^{e^2} \frac{1}{2} x^2 \cdot \frac{1}{x} \, dx \\
&= \left[\frac{1}{2} x^2 \ln x \right]_1^{e^2} - \left[\frac{1}{4} x^2 \right]_1^{e^2} \\
&= \frac{1}{2} e^4 \ln e^2 - \frac{1}{2} 1^2 \cdot \ln 1 - \left[\frac{1}{4} e^4 - \frac{1}{4} \right] \\
&= \frac{1}{2} e^4 \cdot 2 \ln e - 0 - \frac{1}{4} e^4 + \frac{1}{4} \\
&= \frac{3e^4 + 1}{4} && (\ln e = 1) \tag{6}
\end{aligned}$$

$$\begin{aligned}
\text{(ii) shaded area} &= \int_1^{e^2} (2x - x \ln x) dx = \int_1^{e^2} 2x \, dx - \int_1^{e^2} x \ln x \, dx \\
&= \left[x^2 \right]_1^{e^2} - \left(\frac{3e^4 + 1}{4} \right) \\
&= e^4 - 1 - \left(\frac{3e^4 + 1}{4} \right) \\
&= \frac{4e^4 - 4 - 3e^4 - 1}{4} = \frac{e^4 - 5}{4} \tag{3}
\end{aligned}$$

$$\begin{aligned}
8. \text{ (a)} \quad \vec{BC} &= \begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix} \\
l_1 \text{ has equation} \quad \mathbf{r} &= \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix} \tag{2}
\end{aligned}$$

$$\text{(b)} \quad \vec{AD} = \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix}, \quad l_2 \text{ is } \mathbf{r} = \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix} \tag{2}$$

$$\begin{aligned}
\text{(c) At point of intersection} \quad 2 + 7\lambda &= 6 - 2\mu && \dots[\text{A}] \\
4 - \lambda &= 2 + 6\mu && \dots[\text{B}] \\
1 - \lambda &= 0 + 2\mu && \dots[\text{C}]
\end{aligned}$$

$$\text{from [A] and [B]} \quad \lambda = \frac{1}{2} \text{ and } \mu = \frac{1}{4}$$

$$\text{check in [C]} \quad 1 - \frac{1}{2} = 2 \cdot \frac{1}{4}$$

$$l_1 \text{ and } l_2 \text{ intersect at } \left(5\frac{1}{2}, 3\frac{1}{2}, \frac{1}{2} \right). \tag{4}$$

$$\text{(d) we require the angle between } \begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix} \text{ and } \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix}$$

let angle between lines be θ

$$\begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 6 \\ 2 \end{pmatrix} = \sqrt{7^2 + 1^2 + 1^2} \times \sqrt{2^2 + 6^2 + 2^2} \cos \theta$$

$$-14 - 6 - 2 = \sqrt{51} \sqrt{44} \cos \theta$$

$$\cos \theta = \frac{-22}{\sqrt{51} \sqrt{44}} \quad \theta = 117.7^\circ$$

$$\text{acute angle between lines} = 62.3^\circ \quad (1 \text{ d.p.}) \tag{3}$$