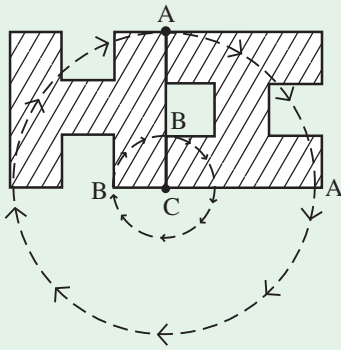


Key Facts



This shape has turned clockwise through a right angle (90° turn or $\frac{1}{4}$ turn).

Each point in the shape rotates around a circle with its centre at the dot (C).

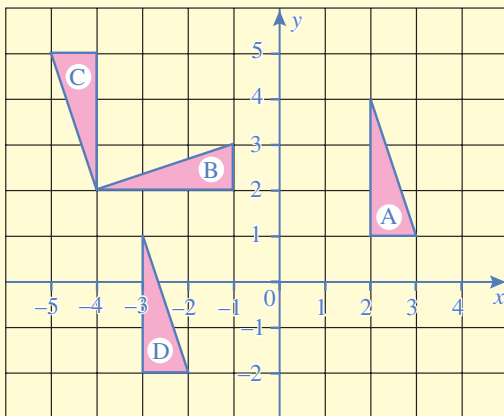
The dot (C) is called the **centre of rotation**.

For 90° rotations, horizontal lines become vertical and vice-versa.

Describing a rotation fully

3 things must be stated:

1. the angle
2. the direction (clockwise or anticlockwise)
3. the centre of rotation



Describe *fully* the rotation which transforms:

- (a) triangle A onto triangle B
- (b) triangle B onto triangle C
- (c) triangle B onto triangle D

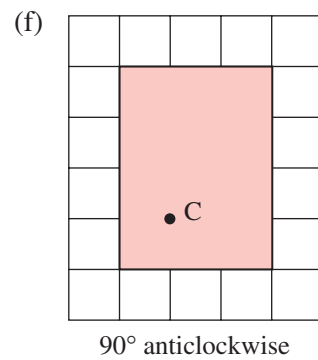
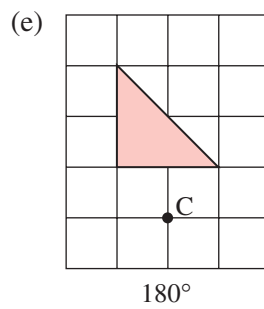
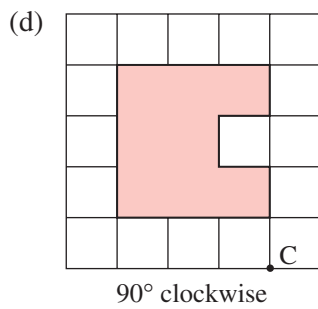
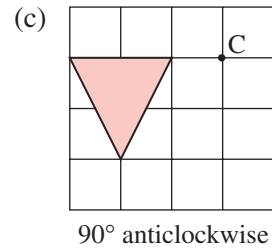
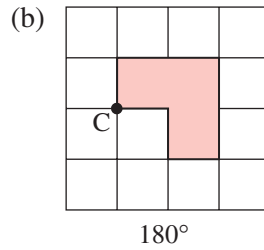
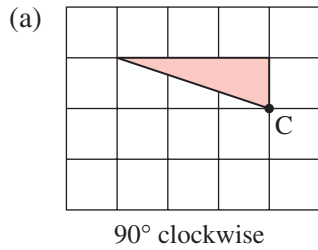
For each answer, we must write down the angle, direction and centre of rotation.

- (a) rotates 90° anticlockwise about $(0, 0)$.
- (b) rotates 90° anticlockwise about $(-4, 2)$.
- (c) rotates 90° clockwise about $(-4, 1)$.

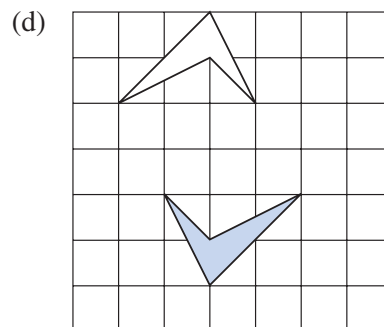
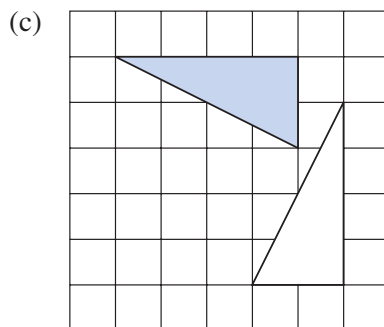
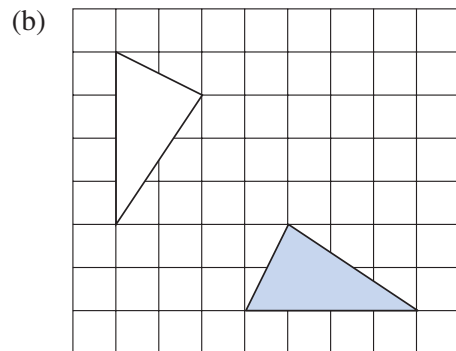
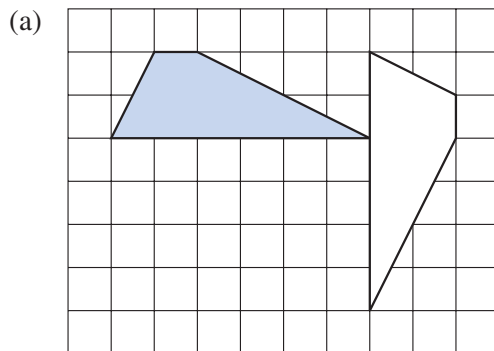
M9.5

You may use tracing paper

- 1 Draw each shape and centre of rotation (C) below. Rotate the shape as indicated and draw the image.

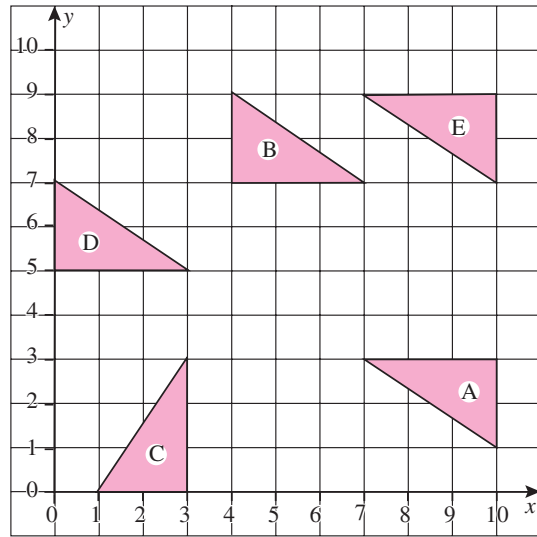


- 2 In each question below the shaded shape rotates onto the other shape. Copy each diagram and mark the centre of rotation.

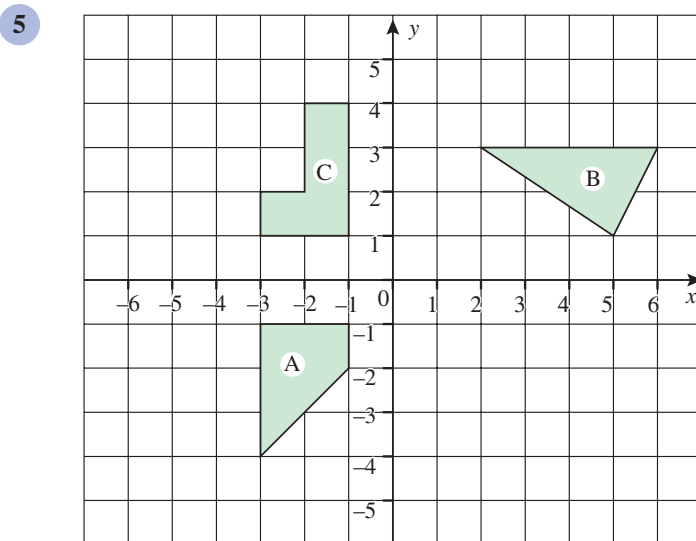


- 3 Find the co-ordinates of the centres of the following rotations:

- (a) triangle A onto triangle B
 (b) triangle A onto triangle C
 (c) triangle A onto triangle D
 (d) triangle C onto triangle E



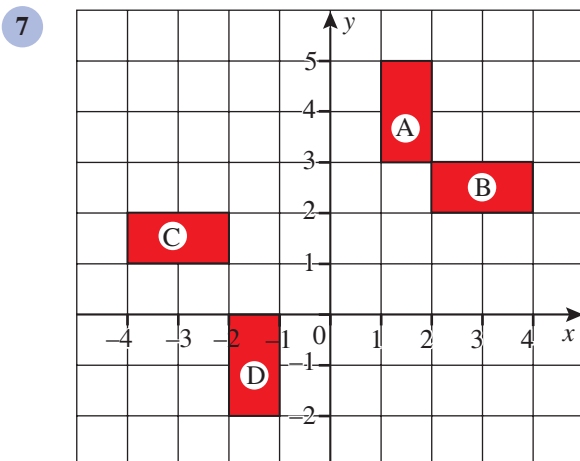
- 4 (a) Draw x and y axes with values from -6 to 6 . Draw triangle A with vertices at $(-5, 2)$, $(-5, 6)$ and $(-3, 5)$.
 (b) Rotate triangle A 90° clockwise about $(-4, -2)$. Label the image B.
 (c) Rotate triangle B 90° clockwise about $(6, 0)$. Label the image C.
 (d) Rotate triangle C 180° about $(1, 1)$. Label the image D.
 (e) Rotate triangle D 90° anticlockwise about $(-5, 1)$. Label the image E.
 (f) Describe *fully* the rotation which transforms triangle E onto triangle A.



Copy the grid and shapes opposite.

- (a) Rotate shape A 90° anticlockwise about $(-3, -4)$. Label the image P.
 (b) Rotate shape B 90° clockwise about $(1, 0)$. Label the image Q.
 (c) Rotate shape C 90° clockwise about $(2, 1)$. Label the image R.

- 6 (a) Draw x and y axes with values from -5 to 5 . Draw rectangle A with vertices at $(0, 2)$, $(0, 5)$, $(-2, 5)$ and $(-2, 2)$.
 (b) Rotate rectangle A 180° about $(-2, 2)$. Label the image B.
 (c) Rotate rectangle B 90° clockwise about $(0, -1)$. Label the image C.
 (d) Rotate rectangle C 180° about $(2, 0)$. Label the image D.
 (e) Rotate rectangle D 90° clockwise about $(3, -2)$. Label the image E.



Describe *fully* the rotation which transforms:

- (a) shape A onto shape B
- (b) B onto C
- (c) C onto D

- 8
- (a) Draw an x -axis from -4 to 4 and a y -axis from -3 to 3 .
Draw triangle A with vertices at $(-1, -2)$, $(-2, -2)$ and $(-2, 0)$.
 - (b) Rotate triangle A 90° clockwise about $(0, -1)$. Label the image B.
 - (c) Rotate triangle B 180° about $(1, 1)$. Label the image C.
 - (d) Reflect triangle C in the x -axis. Label the image D.
 - (e) Reflect triangle D in the line $y = -2$. Label the image E.
 - (f) Translate triangle E through $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$. Label the image F.
 - (g) Describe *fully* the rotation which transforms triangle F onto triangle A.

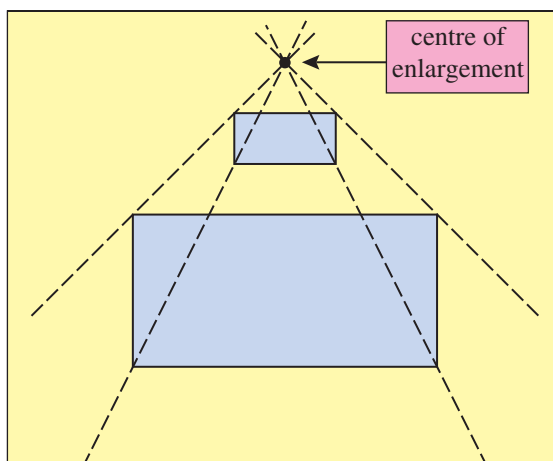
M Enlargement

A mathematical enlargement has a **centre of enlargement** and a **scale factor**.

Every length in the original shape must be multiplied by the scale factor.

This shaded rectangle is enlarged by a scale factor 3 (each length is multiplied by 3).

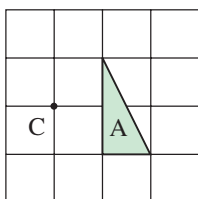
The centre of enlargement is found by drawing a broken line through a corner of the new shape and the same corner of the old shape.



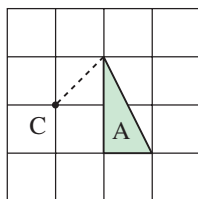
Do this for each pair of points as shown in the diagram. The centre of enlargement is the point where all the broken lines meet (intersect).

Drawing an enlargement

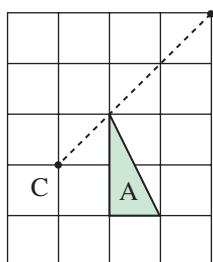
Draw an enlargement of triangle A with scale factor 3 about the centre of enlargement C.



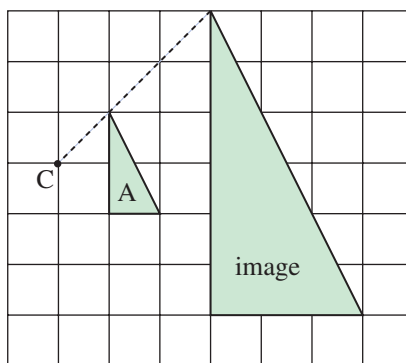
Join the centre C to one vertex with a dotted line



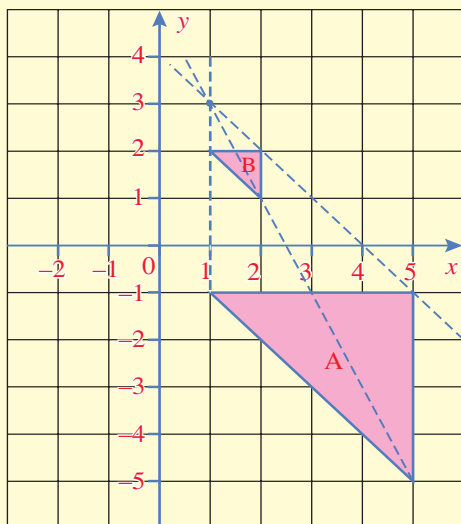
Multiply the length of the dotted line by the scale factor (do this by measuring or counting squares) then draw the longer dotted line from C



This shows where the top vertex will move to



The rest of the enlarged shape can be drawn from this new vertex



Describe fully the enlargement which transforms shape A onto shape B.

Draw broken lines through each corner of the new shape and the same corner of the old shape.

The centre of enlargement is where the broken lines meet (intersect).

Answer:

Enlargement by scale factor $\frac{1}{4}$ about (1, 3)

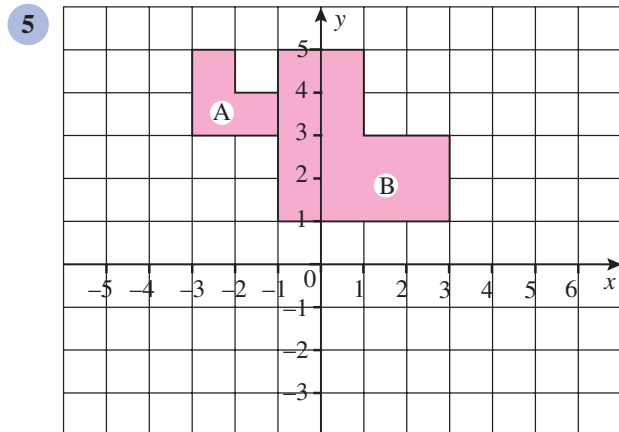
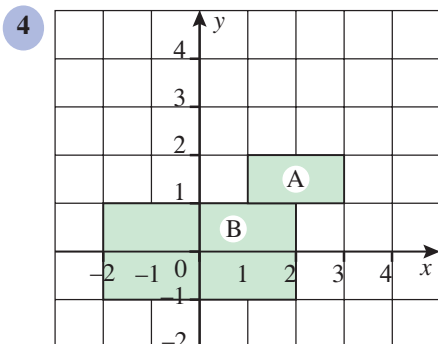
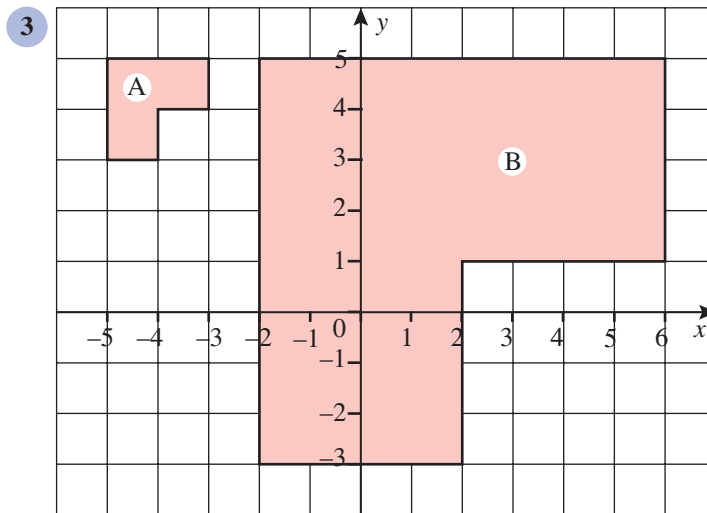
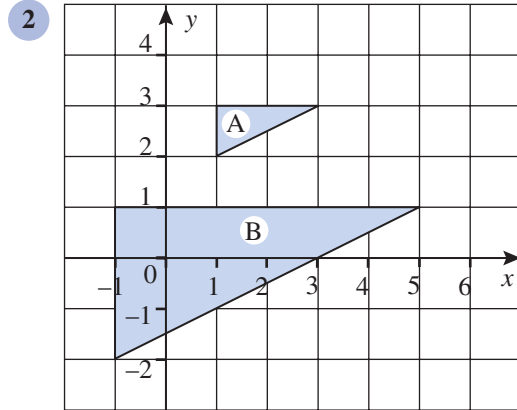
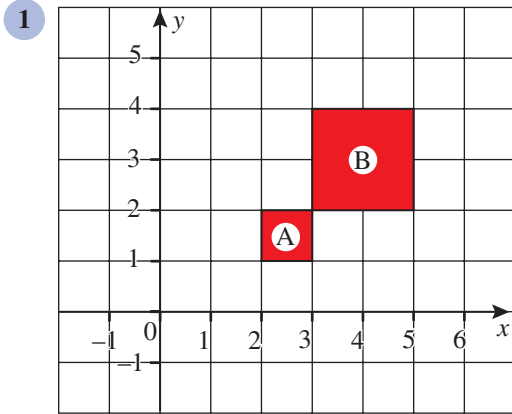
Note

Fractional scale factor

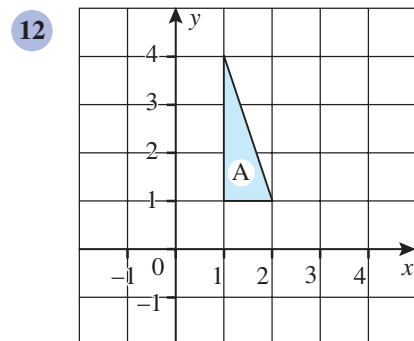
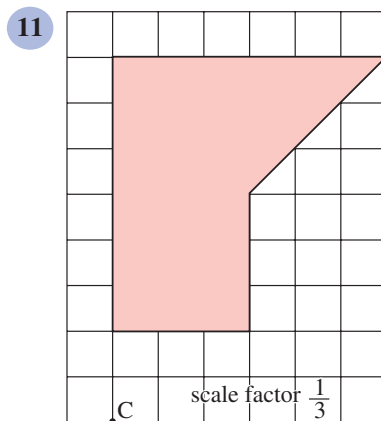
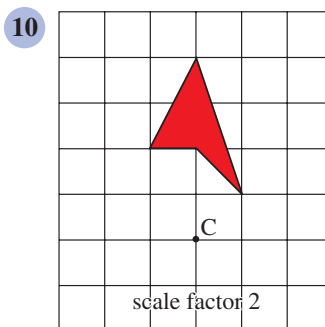
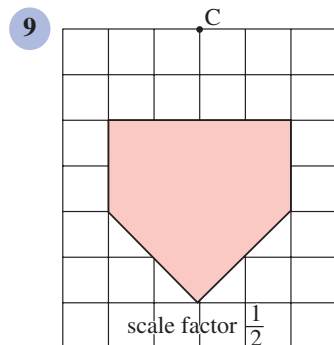
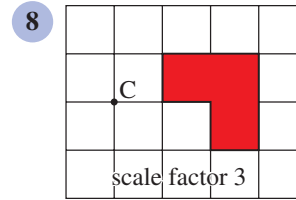
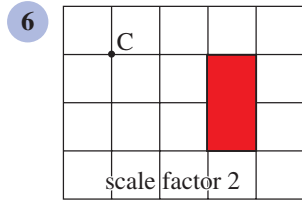
If a shape is *reduced in size*, the scale factor will be a fraction between 0 and 1. It is still known as an enlargement.

M9.6

For Questions 1 to 5, draw the grid and the 2 shapes then draw broken lines through pairs of points in the new shape and the old shape. Describe fully the enlargement which transforms shape A onto shape B.



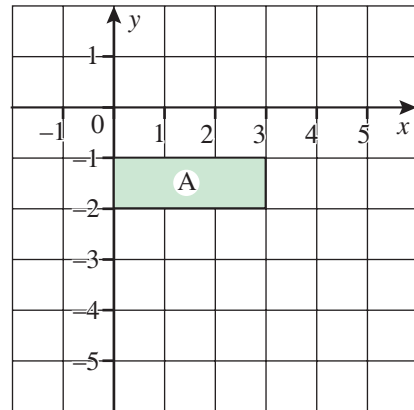
For questions 6 to 11, copy the diagram and then draw an enlargement using the scale factor and centre of enlargement (C) given. Leave room for the enlargement!



Copy the grid and triangle A. Enlarge triangle A by scale factor 2 about (0, 2). Label the image B.

- 13 (a) Draw an x -axis from -2 to 5 and a y -axis from -5 to 2 .
 (b) Draw a square A with vertices $(2, -2)$, $(4, -2)$, $(4, -4)$ and $(2, -4)$.
 (c) Enlarge square A by scale factor $\frac{1}{2}$ about $(0, 0)$. Label the image B.
- 14 (a) Draw an x -axis from -4 to 8 and a y -axis from -4 to 6 .
 (b) Draw a triangle A with vertices at $(1, 3)$, $(1, 4)$ and $(3, 4)$.
 (c) Enlarge triangle A by scale factor 2 about $(1, 5)$. Label the image B.
 (d) Enlarge triangle B by scale factor $\frac{1}{2}$ about $(-3, 1)$. Label the image C.
 (e) Enlarge triangle C by scale factor 3 about $(-2, 3)$. Label the image D.

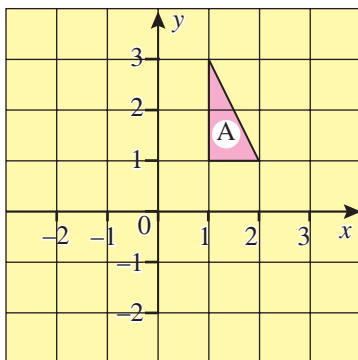
- 15 Copy the grid and rectangle A. Enlarge rectangle A by scale factor 2 about (0, 0). Label the image B.



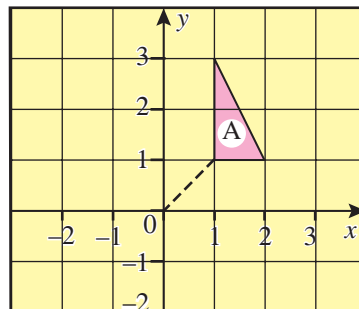
- 16 (a) Draw an x -axis from -6 to 10 and a y -axis from -3 to 5 .
 (b) Draw a shape A with vertices of $(-2, 2)$, $(-2, 3)$, $(-1, 3)$, $(-1, 4)$, $(-4, 4)$, $(-4, 3)$, $(-3, 3)$ and $(-3, 2)$.
 (c) Enlarge shape A by scale factor 3 about $(-6, 4)$. Label the image B.
 (d) Enlarge shape B by scale factor $\frac{1}{3}$ about $(-3, -2)$. Label the image C.
 (e) Enlarge shape C by scale factor $\frac{1}{2}$ about $(0, 0)$. Label the image D.
- 17 (a) Draw an x -axis from -6 to 6 and a y -axis from -7 to 7 .
 (b) Draw a triangle A with vertices of $(2, 2)$, $(2, 6)$ and $(4, 6)$.
 (c) Enlarge triangle A by scale factor $\frac{1}{2}$ about $(0, 0)$. Label the image B.
 (d) Reflect triangle B in the y -axis. Label the image C.
 (e) Enlarge triangle C by scale factor 3 about $(-1, 4)$. Label the image D.
 (f) Rotate triangle D 90° clockwise about $(-1, -5)$. Label the image E.
 (g) Enlarge triangle E by scale factor $\frac{1}{2}$ about $(5, 1)$. Label the image F.

E Negative scale factors

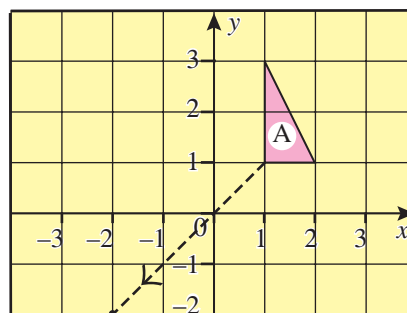
Draw an enlargement of triangle A with scale factor -2 about centre of enlargement at the origin.



- (a) Join the centre of enlargement (in this case the origin) to one vertex with a dotted line.

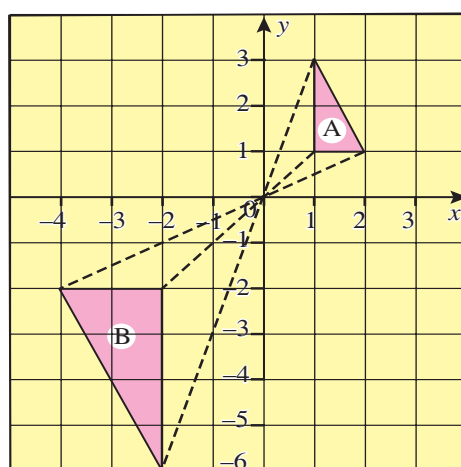


- (b) Multiply the length of the dotted line by the scale factor (i.e. 1 diagonal unit multiplied by -2 gives -2 diagonal units). The negative sign indicates that the longer dotted line should be drawn from the centre in the *opposite direction*.



This shows where the new vertex will move to

- (c) The rest of the enlarged shape can be drawn from this new vertex (more dotted lines can be drawn if necessary).



Key Facts

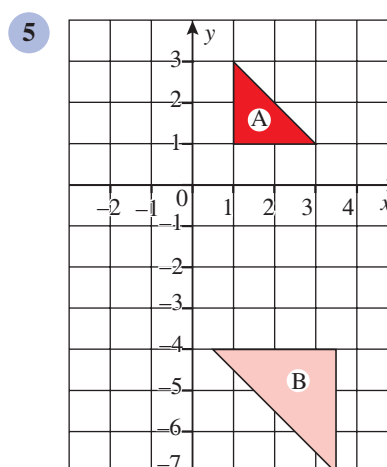
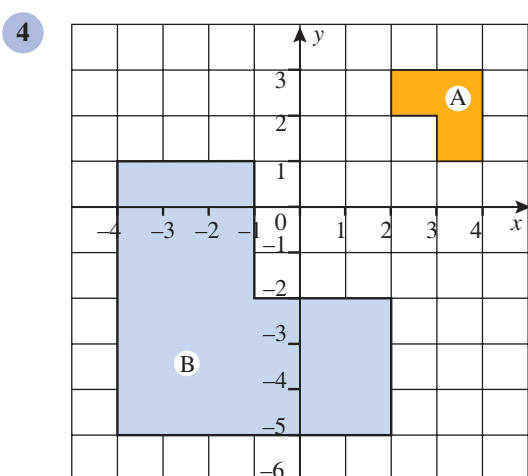
A negative scale factor leads to an image on the opposite side of the centre of enlargement to the original shape (the original shape is turned 'upside down').

E9.1

- 1
 - (a) Draw an x -axis from -1 to 6 and a y -axis from -9 to 5 .
 - (b) Draw a rectangle A with vertices at $(1, 2)$, $(1, 4)$, $(2, 4)$ and $(2, 2)$.
 - (c) Enlarge rectangle A by a scale factor of -3 about $(2, 1)$. Label the image B.
- 2
 - (a) Draw an x -axis from -4 to 4 and y -axis from -5 to 3 .
 - (b) Draw a triangle A with vertices at $(-1, 1)$, $(-1, 2)$ and $(-3, 1)$.
 - (c) Enlarge triangle A by a scale factor of -2 about $(-1, 0)$. Label the image B.
 - (d) Write down the translation vector which describes the movement of the vertex of $(-3, 1)$ in triangle A to its new position in triangle B.

- 3** (a) Draw an x -axis from -4 to 6 and a y -axis from -3 to 4 .
- (b) Draw a triangle A with vertices of $(-2, -1)$, $(-2, -2)$ and $(-3, -1)$.
- (c) Enlarge triangle A by a scale factor of -2 about $(0, 0)$. Label the image B.
- (d) Enlarge triangle B by a scale factor of $-\frac{1}{2}$ about $(2, 0)$. Label the image C.
- (e) Write down the translation vector which will translate triangle C back onto triangle A.

For questions **4** and **5**, describe fully the enlargement which transforms shape A onto shape B (draw the grid and shape if necessary).

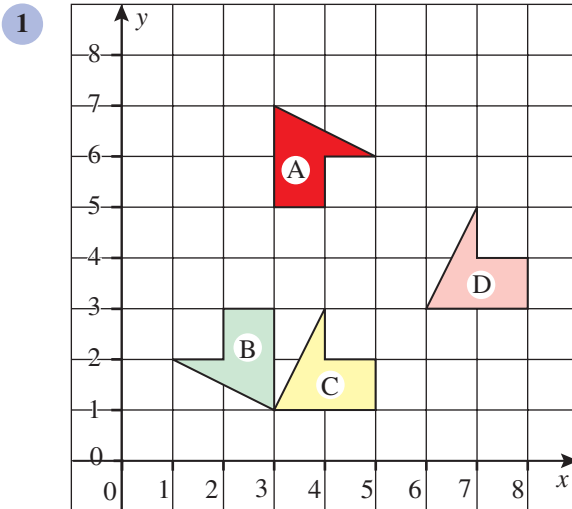


- 6** (a) Draw an x -axis from -5 to 5 and a y -axis from -6 to 3 .
- (b) Draw a triangle A with vertices at $(-2, 2)$, $(-4, 2)$ and $(-3, 1)$.
- (c) Enlarge triangle A by a scale factor of -3 about $(-2, 1)$. Label the image B.
- (d) Enlarge triangle B by a scale factor of $\frac{1}{3}$ about $(1, -5)$. Label the image C.
- (e) Reflect triangle C in the line $x = -1$. Label the image D.
- (f) Describe where triangle D would move to if it was enlarged by a scale factor of -1 about $(-3, -1)$.
- 7** (a) Draw an x -axis from -7 to 4 and a y -axis from -5 to 9 .
- (b) Draw a shape A with vertices at $(4, 1)$, $(4, 3)$, $(2, 3)$, $(2, 5)$, $(0, 5)$ and $(0, 1)$.
- (c) Enlarge shape A by a scale factor of $-\frac{1}{2}$ about $(-2, -1)$. Label the image B.
- (d) Enlarge shape B by a scale factor of -3 about $(-4, -1)$. Label the image C.
- (e) Enlarge shape C by a scale factor of $\frac{2}{3}$ about $(-4, -1)$. Label the image D.
- (f) Write down the translation vector which will return shape D to shape A.

E9.2

The questions in this Exercise use a mixture of transformations.

You may use tracing paper.

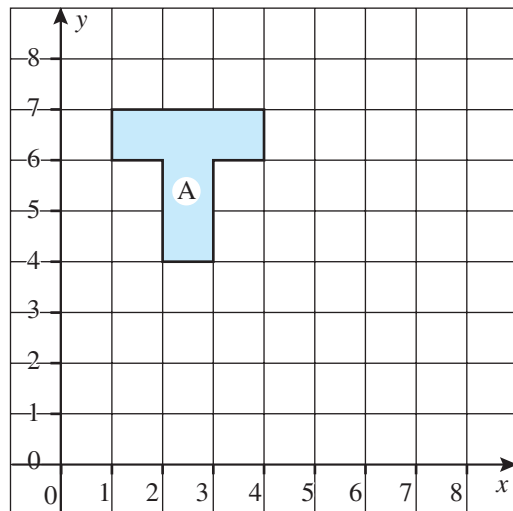


Describe *fully* the transformation which maps (sends):

- A onto B
- B onto C
- C onto D

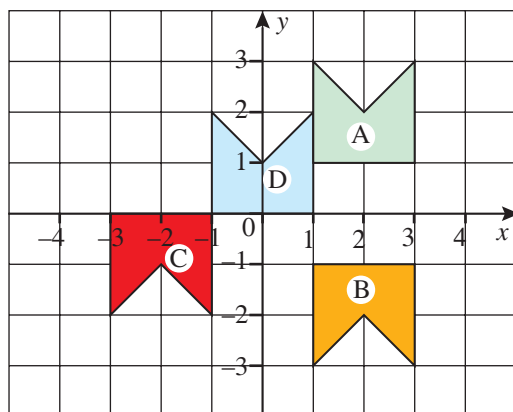
2 Copy the grid and shape opposite.

- Rotate shape A 90° clockwise about (2, 3). Label the image B.
- Rotate shape B 90° anticlockwise about (6, 4). Label the image C.
- Describe *fully* the translation which maps shape C onto shape A.

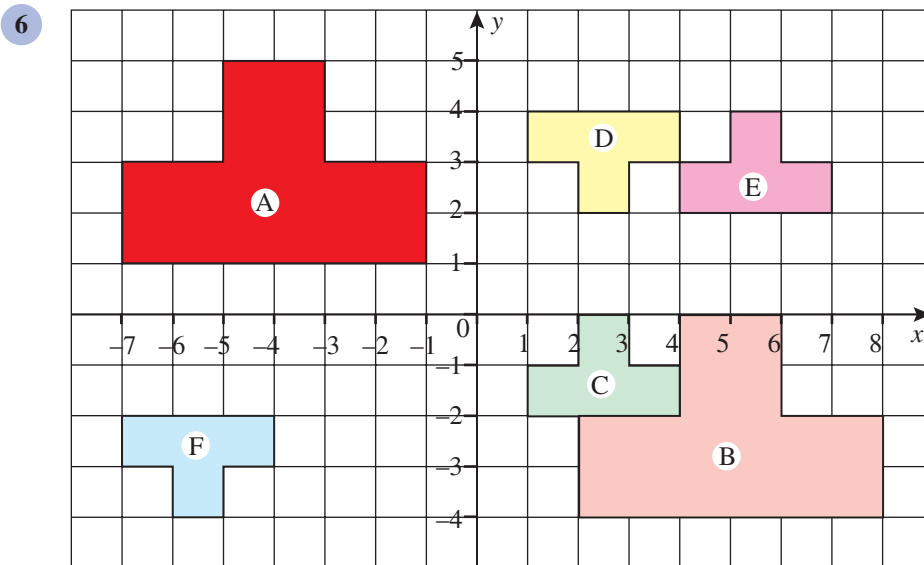


3 Describe *fully* the transformation which maps:

- A onto B
- B onto C
- C onto D
- D onto A



- 4 (a) Draw an x -axis from -5 to 10 and a y -axis from -8 to 5 .
 (b) Draw a shape A with vertices at $(2, 2)$, $(2, 4)$, $(3, 3)$, $(5, 3)$ and $(5, 2)$.
 (c) Rotate shape A 180° about $(3, 1)$. Label the image B.
 (d) Enlarge shape B by scale factor 3 about $(1, 1)$. Label the image C.
 (e) Reflect shape B in the y -axis. Label the image D.
 (f) Reflect shape D in the line $y = 1$. Label the image E.
 (g) Describe *fully* the transformation which maps shape E onto shape A.
- 5 (a) Draw an x -axis from -6 to 8 and a y -axis from -6 to 10 .
 (b) Draw a triangle P with vertices at $(0, -1)$, $(2, -1)$ and $(1, 2)$.
 (c) Translate triangle P through $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$. Label the image Q.
 (d) Enlarge triangle P by scale factor 2 about $(-1, -2)$. Label the image R.
 (e) Reflect triangle R in the line $y = x + 3$. Label the image S.
 (f) Rotate triangle S 90° clockwise about $(-2, -1)$. Label the image T.



Describe *fully* the transformation which maps:

- (a) A onto B
 (b) B onto C
 (c) C onto D
 (d) D onto E
 (e) E onto F
- 7 (a) Draw x and y -axes from -8 to 8 .
 (b) Draw a rectangle A with vertices at $(2, 2)$, $(6, 2)$, $(6, 4)$ and $(2, 4)$.

- (c) Draw a rectangle B with vertices at $(-4, 4)$, $(-4, 0)$, $(-2, 0)$ and $(-2, 4)$
- (d) Describe *fully* the transformation which maps A onto B
- (e) Reflect rectangle A in the line $y = -x$. Label the image C.
- (f) Draw a rectangle D with vertices at $(8, -4)$, $(0, -4)$, $(0, -8)$ and $(8, -8)$.
- (g) Describe *fully* the transformation which maps A onto D (the vertex $(2, 2)$ in rectangle A is now at $(8, -4)$ in rectangle B).
- (h) Rotate rectangle C 90° anticlockwise about $(-5, 5)$. Label the image E.
- (i) What is the equation of the line of reflection which maps rectangle E onto rectangle A?

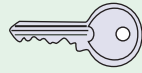
- 8**
- (a) Draw an x -axis from -8 to 8 and a y -axis from -8 to 12 .
 - (b) Draw a triangle P with vertices at $(5, 3)$, $(7, 4)$ and $(6, 6)$.
 - (c) Enlarge triangle P by a scale factor of -1 about $(3, 5)$. Label the image Q.
 - (d) Enlarge triangle P by a scale factor of -2 about $(4, 3)$. Label the image R.
 - (e) Describe *fully* the transformation which maps triangle R onto triangle Q.
 - (f) Rotate triangle P 90° clockwise about $(2, 0)$. Label the image S.
 - (g) Draw a triangle T with vertices at $(-2, -6)$, $(-1, -8)$ and $(1, -7)$.
 - (h) Describe *fully* the rotation which maps triangle P onto triangle T.
 - (i) Describe *fully* the transformation which maps triangle T onto triangle S.

Can you still?

9C Use function notation (see Unit 6)

Can you still?

1. If $f(x) = 9x - 5$, find the value of (a) $f(4)$ (b) $f(-2)$
2. If $g(x) = x^3 - x$, find the value of (a) $g(4)$ (b) $g(-1)$
3. If $p(x) = (2x + 1)^2$, find the value of (a) $p(5)$ (b) $p\left(\frac{-1}{2}\right)$
4. If $s(x) = \frac{x^2 + x}{5x - 2}$, find the value of (a) $s(1)$ (b) $s(-2)$
5. If $h(x) = 7(x - 3)$, find the value of x when $h(x) = 21$.
6. If $g(x) = x^2 - 5x + 6$, find the values of x when $g(x) = 0$.
7. If $f(x) = x^2 - 8x$, find the value of z when $f(z) = 2z$.
8. If $g(x) = \frac{5 - 2x}{3x}$, find the value of w when $g(w) = 1$.
9. Write down the expression for $f(2x)$ if $f(x) = x^2$.
10. Write down the expression for $f(x - 3)$ if $f(x) = 5x + 4$.



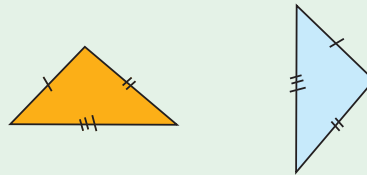
Key Facts

Two shapes are congruent if they are exactly the same size and shape.

There are four ways of proving that two triangles are congruent:

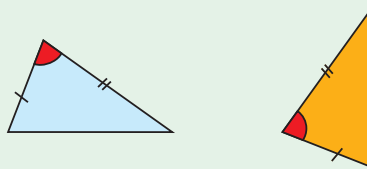
1. SSS (side, side, side)

All 3 sides are equal.



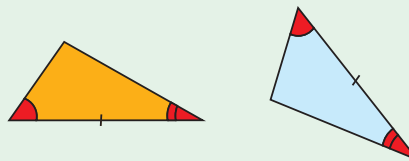
2. SAS (side, angle, side)

Two sides and the angle between them are equal.



3. AAS (angle, angle, side)

Two angles and a corresponding side are equal.

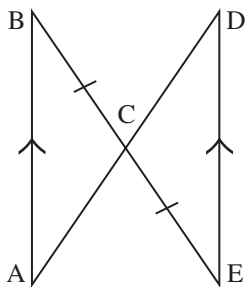


4. RHS (right angle, hypotenuse, side)

Each triangle has a right angle. The hypotenuse and one other side are equal.



Example



AB is parallel to ED. C is the midpoint of BE.

Prove that $AC = CD$.

$\hat{A}BC = \hat{C}ED$ (alternate angles)
 $\hat{B}CA = \hat{E}CD$ (vertically opposite angles)
 $BC = CE$ (C is midpoint of BE)

3 conditions AAS proves congruence

proof requires reasons to be given

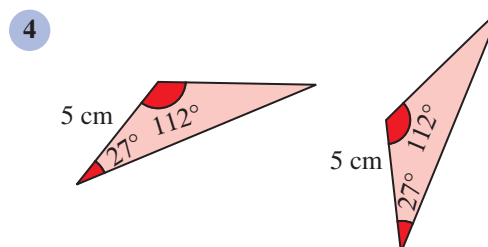
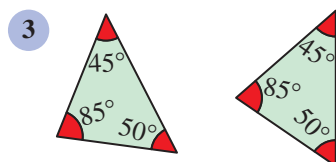
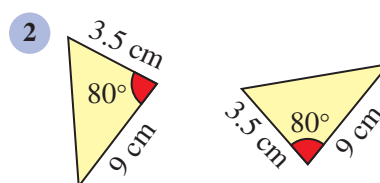
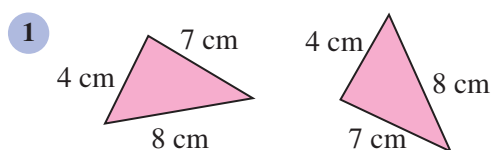
so Δ 's $\begin{matrix} ABC \\ DEC \end{matrix}$ are congruent (AAS)

write the triangles underneath each other so that each angle corresponds to its equal angle in the other triangle

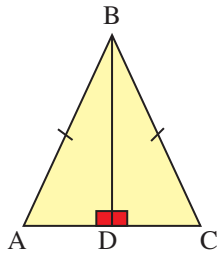
so $AC = CD$ because the triangles are congruent (we can see that the letters A and C correspond to the letters D and C in the 'congruent triangles' statement above).

E9.3

In questions 1 to 4, state whether the pair of triangles are congruent. Give the reason (eg. SAS) if they are congruent.



5



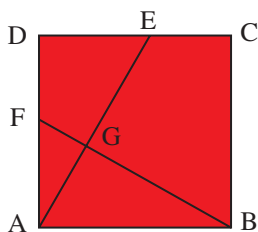
ABC is an isosceles triangle.

- Prove that $\triangle ABD$ is congruent to $\triangle CBD$.
- Explain why $\hat{A}BD = \hat{C}BD$ (this proves that the perpendicular bisector BD cuts $\hat{A}BC$ in half).

6

Draw a quadrilateral ABCD in which $AB = AD$ and $BC = CD$ (ie. a kite). Draw the line AC and state which two triangles are congruent. Hence prove that the angle at B is equal to the angle at D.

7

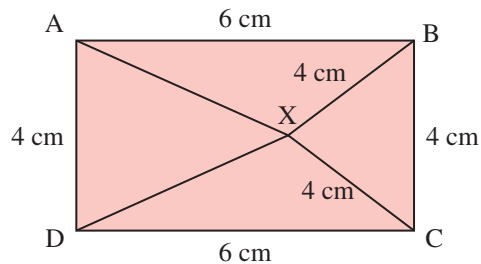


ABCD is a square. E is the midpoint of DC and F is the midpoint of AD. Prove that triangles ADE and BAF are congruent.

8

ABCD is a rectangle.

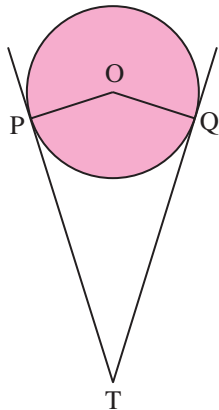
- Prove that triangles ABX and DCX are congruent.
- Hence prove that DXA is an isosceles triangle.



9

Draw a quadrilateral ABCD (not a rectangle) such that AB is parallel to CD and AD is parallel to CB. Draw the line AC. Use congruent triangles to prove that opposite sides of this shape are equal.

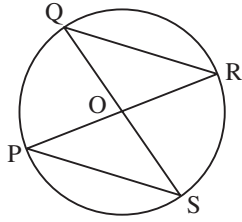
10



PT and QT are tangents to the circle with centre O.

- Find \hat{OQT} and \hat{OPT} .
- Explain why OP is equal to OQ.
- Hence state, with reasons, a triangle which is congruent to triangle PTO.

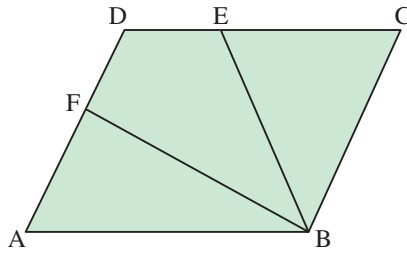
11



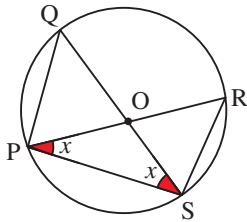
Prove that Δ s OQR and OPS are congruent.

12

In the diagram below ABCD is a rhombus, E is the midpoint of DC and F is the midpoint of AD. Show that triangles AFB and ECB are congruent.

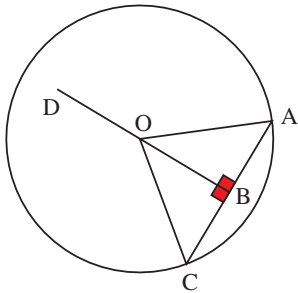


13



O is the centre of the circle. State, with reasons, which triangle is congruent to PRS.

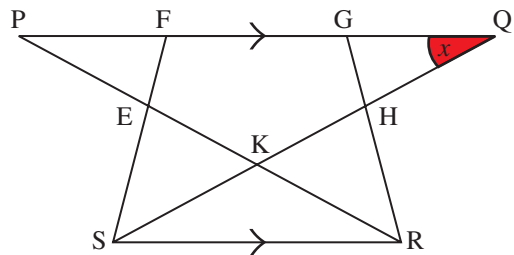
14



O is the centre of the circle. Use congruent triangles to prove that the perpendicular line (DB) to the chord (AC) bisects the chord AC.

15

In the diagram, $PK = QK$, $PF = QG$ and the lines PQ and SR are parallel. The angle PQS is x .



(a) Find, with reasons, the following angles:

(i) $\hat{Q}PR$

(ii) $\hat{Q}SR$

(iii) $\hat{P}RS$

(b) Explain why $SK = RK$.

(c) Hence show that $PR = QS$.

(d) State, with reasons, which triangle is congruent to QSF.

USE YOUR MATHS! – Student finance

If you choose to go to University/College, it will cost you money. Your main costs will be:

- tuition fees (the cost of your course)
- accommodation (where you live)
- living expenses (eg. food, bills, books and going out)

How do I manage?

The main types of financial help available are:

- A student loan for tuition fees (up to £3145 per year in 2008/09)
- A maintenance grant – depends on parents' household income (worth up to £2835 a year in 2008/09)
- A student loan (up to £4625 a year in 2008/09 if you live away from home and £6475 in London) – all students are entitled to 75% of the loan but the remaining 25% is means-tested
- A bursary from your university or college.



This table shows how much maintenance grant you could receive in 2008/09.

You do not have to pay back any maintenance grant you get.

Household income	Maintenance grant
Up to £25000	£2835
Between £25001 and £34450	£2834 to £1260. Reduced by £1 for every £6 of income above £25000
Between £34451 and £60005	£1259 to £50. Reduced by £1 for every £21.12 of income above £34450
Over £60005	Nil

Task A

Work out the possible maintenance grant for each of the people below with the household income as indicated.

John, household income £27568; Amy, household income £51113; Callum, household income £63102 ; Tania, household income £32680

Repaying student loans

When you have finished your course, you must start repaying the loan if you get a job earning over £15000.

You will repay with 9% of your earnings over £15000 but can repay more if you want to. The first repayment will be due at the end of April after you leave your course.

Example : if you earn £23500 a year, you will pay 9% of £8500 (£23500–£15000). This would be around £14.71 each week.

The dreaded interest!

The amount you owe on your student loan increases by 1.5% each year (at the time of writing).

Task B

- 1 David finishes his course with a student loan of £23045 and starts a job in September, earning £25470 per year. Exactly one year later he gets a 4% pay rise.
 - (a) How much of his student loan does David repay in the first year that he works?
 - (b) How much of his student loan does he repay in the second year that he works?
 - (c) If the interest is added to his student loan at the end of April each year, how much will David owe after he has worked for exactly two years?



- 2 Sunita plans to do a History of Art course at Bristol University which will last for 3 years. The course fee will be £3145 in the first year. The course fees increase by 2% in each of the following years. Sunita gets no grant and a student loan of £3470 each year.
 - (a) How much will Sunita owe at the end of her course?
 - (b) She does not get a job until two years after she finishes her course. How much does she now owe for her student loan?
- 3 RESEARCH : choose a University/College town or city. Find out the likely cost of accommodation. Estimate other weekly living costs. Assuming the course lasts for 34 weeks each year, how much do you think you would need for your accommodation and living expenses for one year?