

Example 6

Find the values of the scalars p and q if:

$$\begin{aligned} p\mathbf{i} + q\mathbf{j} &= 3(2\mathbf{i} - \mathbf{j}) + 4\mathbf{i} + \mathbf{j} \\ p\mathbf{i} + q\mathbf{j} &= 6\mathbf{i} - 3\mathbf{j} + 4\mathbf{i} + \mathbf{j} \\ &= 10\mathbf{i} - 2\mathbf{j} \end{aligned}$$

Equating the \mathbf{i} and \mathbf{j} components:

$$\begin{aligned} p &= 10 \\ q &= -2 \end{aligned}$$

2.7 Parallel vectors

Vectors \mathbf{a} and \mathbf{b} are parallel if:

$k\mathbf{a} = \mathbf{b}$ where k is a constant.

In other words, if you can multiply one vector by a constant to get the other vector, then those vectors are parallel.

Example 7

Vectors

$$\begin{aligned} \mathbf{a} &= 2\mathbf{i} + \mathbf{j} \\ \mathbf{b} &= 4\mathbf{i} + 2\mathbf{j} \\ \mathbf{c} &= 6\mathbf{i} + 3\mathbf{j} \end{aligned} \quad \text{are all parallel.}$$

2.8 Finding a unit vector parallel to a given vector

Example 8

Find the unit vector which is parallel to the vector $5\mathbf{i} + 12\mathbf{j}$.

The vector $5\mathbf{i} + 12\mathbf{j}$ has magnitude $\sqrt{5^2 + 12^2} = 13$ units.

So the vector $\frac{5\mathbf{j} + 12\mathbf{j}}{13}$ will be parallel to $5\mathbf{i} + 12\mathbf{j}$ and will be of unit length.

(A 'unit vector' is a vector of length 1 unit.)

2.9 Obtaining a vector given its magnitude and direction

Example 9

The vector \mathbf{v} has magnitude 20 and is in the direction of the vector $3\mathbf{i} + 4\mathbf{j}$. Find \mathbf{v} in the form $p\mathbf{i} + q\mathbf{j}$.

The vector $3\mathbf{i} + 4\mathbf{j}$ has magnitude $\sqrt{3^2 + 4^2} = 5$ units.

So the vector $\frac{3\mathbf{i} + 4\mathbf{j}}{5}$ is a *unit* vector parallel to $3\mathbf{i} + 4\mathbf{j}$.